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Asymmetry of Long-Short Cost in Derivatives Market, Heterogeneous Beliefs and Stock Price Crash: A Theoretical Model

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Abstract: In this paper, we construct a four-period-double-market model in this paper. By including the stock market with short selling restrictions and the derivative market without short selling restrictions but with long-short costs in the model, we study the relationship between the asymmetry of long-short cost in derivative market, investors' heterogeneous beliefs and the stock price crash risk. According to the conclusion of closed solution of our model, the asymmetry of short cost in derivatives market will distort the implied price of derivatives market, which will send a wrong message to stock market and intertwined with investors' heterogeneous beliefs in the stock market. Moreover, under the general equilibrium model, a derivative market with symmetrical long-short cost can completely eliminate the risk of stock price crash. But if the short-selling cost is relative higher than the buying cost, the stock price will be overvalued in the early periods, and the negative events will result in a more serious stock price crash than the single market situation. Our model emphasizes the distorting effect of long-short cost asymmetry on the price discovery and information flow function of derivatives market, and reminds government departments to improve market mechanism and strengthen supervision when promoting the development of derivatives market. The government should actively guide the derivatives market to play its due role in the financial market.

Keywords: Heterogeneous Beliefs, Stock Price Crash, Asymmetric Long-Short Costs

1. Introduction

Research on the causes of stock price collapse has been quite rich. On the one hand, researchers show that the basic characters of enterprises can lead to the increase of the stock crash risk [1-5], management's characteristics and behavior are also one of the main culprits of stock price crash [6-11]. On the other hand, from the perspective of imperfection of the market, researchers demonstrate the short-selling restrictions in the stock market [12-15] and the defects of derivatives market [16, 17] will also increase the stock crash risk. However, these studies are mainly from the perspective of empirical research. There are still few theoretical models on the causes of stock price crash.

A landmark work on the theory of stock price collapse is Hong and Stein (2003) [13]. In this paper, they use a four-period single market model to prove that when there is short-selling restriction in the market, the incomplete, delayed and asymmetric reaction of investors' heterogeneous beliefs leads to the negative information or emotions of investors being hidden first, and then suddenly erupts when bad news occurs, which directly leads to the occurrence of stock price crash.

With the development of financial derivatives market (such as options, futures, CDS market), their characteristics of high leverage, high liquidity, no short-selling restrictions attracts a large number of investors to participate in, and investors' information will inevitably be reflected in these derivatives markets. Since the biggest difference between derivatives market and stock market is that there is no short selling restriction in derivative market, it is very meaningful to consider how investors' heterogeneous beliefs flow in a stock market with short selling restriction and a derivatives market without short selling restriction.

We extend Hong and Stein's (2003) [13] model to a two-market situation. In our model, a stock market with short selling restrictions coexists with a derivative market without short selling restrictions. The cost of long or short stocks in the derivative market will affect the release of investors` information. This information interacts with the information expressed in the stock market, and ultimately, it will have an impact on the stock market crash.

Based on our model, we find that the asymmetry of long-short cost in derivatives market will affect the formation of stock price crash in the final stock market. The existence of short-selling costs distorts the reaction of positive and negative information. Higher short-selling costs are not conducive to the disclosure of negative information and the implied price of derivatives market will be overvalued. This information will lead to excessive price rises in the stock market, so the risk of stock price crashnow is even exceeding that of single market. This phenomenon has been alleviated with the gradual decrease of short-selling costs. In particular, the existence of derivatives market can completely eliminate the risk of stock price collapse when there are symmetrical long-short costs (that is, when the cost of buying is equal to the cost of short selling), in another word, the information is perfectly disclosed at this time and the equilibrium stock price perfectly reflects the heterogeneous beliefs of all investors.

2. Economic Setup

2.1. Basic Setting

Based on Hong and Stein (2003) [13], the model has 4 dates, which can be label time 0, 1, 2 and 3. There is a stock that will pay a terminal dividend of D on the stock market at time 3. Unlike Hong and Stein (2003) [13], which only considers a stock market with short selling constraints, we introduce a derivatives markets to relax the short selling constraints in the stock market. More generally, we consider the transaction costs of longing or shorting a stock in derivatives markets, and we find that the asymmetric trading costs will affect the final stock price and ultimately affect the stock price crash. Last but not least, we assume that there are 3 kinds of investors, including an optimistic investor A, a pessimistic investor B and risk-neutral rational arbitrage investors. The optimistic investor A will get a good signal about the stock dividend, while the pessimistic investor B will get a bad signal about the stock dividend. Both A and B cannot short their stock in the stock market because of the short sell constraint, but they can short or long the stock in the derivative market after they pay the transaction costs. The arbitrageurs can be treated as the market-maker, they have no access to information and the main job of them is to clear the market and set a rational price. As the market-maker, arbitrageurs can short their stock in the stock market and derivatives market without any cost.

2.2. Information Structure and Investor Demand

Investor A and B take turns getting signals about the terminal dividend. In particular, at time 1, investor B get S_B, then investor A will get S_A at time 2. We think each investor is equally informative and the terminal dividend is given by

$$\mathrm{D} = \frac{S_A+S_B}{2} + \varepsilon, \varepsilon\sim N(0,\sigma^2) \tag{1}$$

S_B is uniformly distributed on the interval $[0,2\mathrm{V}]$ and S_A is uniformly and independently distributed on the interval $[\mathrm{H}, 2\mathrm{V}+\mathrm{H}]$. Thus the rational expectation of S_B and S_A are V and $\mathrm{V}+\mathrm{H}$. Here, $\mathrm{H} \in [0,2\mathrm{V}]$ is the ex-ante measure of the heterogeneity of opinions. This set up implies that investor B is more bearish than investor A at the beginning. Also, by restricting the model to the case that B move first can high-light the central intuition that the bad information may hidden first, and can greatly reduce the complexity of the analysis.

Both investors are assumed to be overconfidence, that is, investors believe their own information even after they have known each other`s information. Investors` demand on the stock market in time 1 and time 2, after seeing the stock price P_t andin light of the short-sell constraint, are given by follow.

Investor A`s demand (get information at time 2):

$$Q_A^{S,1} = \begin{cases} V+H-P_1 \; if \; P_1 < V+H: buy \; when \; price \; is \; low \\ 0 \; if \; P_1 \geq V+H: can \; not \; short \; sell \; in \; stock \; market \end{cases}$$

$$Q_A^{S,2} = \begin{cases} S_A-P_2 \; if \; P_2 < S_A: buy \; when \; price \; is \; low \\ 0 \; if \; P_2 \geq S_A: can \; not \; short \; sell \; in \; stock \; market \end{cases} \tag{2}$$

Investor B`s demand (get information at time 1):

$$Q_B^{S,t=1,2} = \begin{cases} S_B-P_t \; if \; P_t < S_B: buy \; when \; price \; is \; low \\ 0 \; if \; P_t \geq S_B: can \; not \; short \; sell \; in \; stock \; market \end{cases} \tag{3}$$

Also, investors can long or short the stock based derivatives on the derivative market, with no short-sell constraint. More generally, we assume there is a transaction cost in the derivative market. Such transaction cost may come from the entry barriers and the margin system. The existence of transaction costs makes it impossible for us to do long or short perfectly. If investors are trying to long (short) N stocks on the derivative market, they can only long c_1N (short c_2N, $c_1, c_2 \in [0,1]$) stocks at last. Here $1-c_1$ and $1-c_2$ measure the cost of long and short, respectively. We can derive the investors` implied demand of stock in the derivative market in time 1 and time 2, after seeing the stock price P_t and in light of the transaction cost,

Investor A`s demand (get information at time 2):

$$Q_A^{O,1} = \begin{cases} c_1(V+H-P_1) \; if \; P_1 < V+H: buy \; when \; price \; is \; low \\ c_2(V+H-P_1) < 0 \; if \; P_1 \geq V+H: short \; sell \; in \; option \; market \end{cases}$$

$$Q_A^{O,2} = \begin{cases} c_1(S_A - P_2)\ if\ P_2 < S_A: buy\ when\ price\ is\ low \\ c_2(S_A - P_2) < 0\ if\ P_2 \geq S_A: short\ sell\ in\ option\ market \end{cases} \tag{4}$$

Investor B`s demand (get information at time 1):

$$Q_B^{O,t=1,2} = \begin{cases} c_1(S_B - P_t)\ if\ P_t < S_B: buy\ when\ price\ is\ low \\ c_2(S_B - P_t) if\ P_t \geq S_B: short\ sell\ in\ option\ market \end{cases} \tag{5}$$

2.3. The Price-Setting Mechanism

At time 0, no investor gets the information and the initial price will be set as theex-ante expectation of the terminal dividend

$$P_0^S = \frac{V+H+V}{2} = V + \frac{H}{2} \tag{6}$$

As time goes on, more information will be realized and the pricing process become more complicated, we follow the mechanism of Hong and Stein (2003) [13].

This pricing is similar to a down price auction, consider an auctioneer who will charge a trial price p_t and any time t and investors respond by calling out their demand. Because of the short-sell constraint, A and B can only call out something when their demand is positive, while the arbitrageurs are free to call out either positive or negative demands. The arbitrageurs are able to observe any demands called out by A and B and the market clears when the arbitrageurs` demand are 0. The auction will lower the price after seeing a negative demand from the arbitrageurs and raises the price when the arbitrageurs` demand is positive. This process continues until the market clears. Then, investor A and B will long or short the stock-based derivatives in the derivative market and the price adjustment process in the derivative market is given by follow.

Given a stock price P_s and the signal of investor A and B: S_A and S_B. Generally, we assume $S_A > P^S > S_B$. Thus, investor A will long the stock in the derivative market while investor B will short the stock and the implied stock price on the derivative market is

$$P^O = P^S + c_1(S_A - P^S) + c_2(S_B - P^S) \tag{7}$$

Then the arbitrageurs adjust the stock price based on the derivative market, we assume that the final price will be the weighted average of the stock price and the implied stock price on the derivative market

$$P^F = (1-\theta)P^S + \theta P^O = P^S + \theta(c_1(S_A - P^S) + c_2(S_B - P^S)) \tag{8}$$

Here, θ, the weight of the derivative implied price, measures the quality of information transmitted from the derivative market to the stock market and the arbitrageurs` dependence on derivatives market information

Lemma 1. If the bad information S_B is hidden at price P^S ($S_B \leq P^S$), such information will keep hidden in the stock market after the adjustment of the derivative market.

An intuitive property of the pricing process is that the bad information may not initially appear in the stock market because of the auction process, but with the derivatives market adjustment, the final price may be more reasonable than the single market.

2.4. A Simple Example

The following story is a simple example of our model. Consider a stock that will pay a dividend of \$8 at time 3. Investor A and B will receive different information at different times. In particular, at time 1, investor B get S_B, then investor A will get S_A at time 2. We assume that S_B is uniformly distributed on the interval $[0,20]$ and S_A is uniformly and independently distributed on the interval $[5,20+5]$, here 5 measures the heterogeneous believe between investor A and B. In other words, this is difference between the optimistic opinion and pessimistic opinion.

At time 0, the optimistic investor A and the pessimistic investor B share different expectations for the dividend, \$15 and \$10. If the risk-neutral rational arbitrage investors know all the information about A and B, then the reasonable price given by arbitrageurs is the mean of the expected price of A and B, \$12.5.

At time 1, the pessimistic investor B will get bad news about the terminal dividend, say \$3, while the optimistic investor A gets no new information, the rational price at this case should be (\$3+\$15)/2=\$9. Now let`s first consider the stock market. Investor A will buy the stock as long as the price is less than \$12, but investor B will keep silence when the price is greater than \$6 because of the short sell constraint. The arbitrageurs, with no short sell constraint, will clear the market and the equilibrium price is the price when the demand for arbitrageurs is 0, that is to say, the equilibrium price will be reached when the market price is equal to the expected price of arbitrageurs. For example，when investors face a price of \$12, investor A will long the stock but investor B will do nothing, thus the arbitrageurs see A`s signal \$15 and give an expectation of B`s signal which equal to half the current price \$12/2=\$6. And their expected price at this time is (\$6+\$15)/2=\$10.5 which is less than the current price \$12. The arbitrageurs are willing to long the stock and the stock price will go down. In fact, according to the proof of Lemma 2 (see Appendix), the equilibrium stock price in this case should be \$10. At this price, A will long the stock and B won`t do anything, the arbitrageurs` expectation of B`s signal is \$5, which is higher than the real value of B`s signal, that is, the bad news seems to be hidden in this case. Arbitrageurs` expected stock price is (\$5+\$15)/2=\$10, which is equal to the current price and this is the equilibrium state in the stock market. Now let`s look at the derivative market. Since A believes the stock price should be \$15, A will buy \$15-\$10=\$5 stock in the derivative market, or, to be exact, A will take a long position of the derivative of the stock, and the implied stock value of the derivative position are from \$10 to \$15. As for investor B, he will short \$10-\$3=\$7 stocks in the derivative market. The net position from investor A and B will be a short position of \$2. The arbitrageurs will clear the market and the implied stock price in the derivative market will be \$10-\$2=\$8. Right now, arbitrageurs have different price information in the stock market and derivatives market. We assume that the arbitrageurs have the same confidence in the two markets, thus the final price given by the arbitrageurs is the mean of the two-market price, (\$10+\$8)/2 =\$9, which is equal to the rational price.

At time 2, the optimistic investor A gets good news about the terminal dividend, say \$18. The rational price now is (\$18+\$3)/2 =\$10.5. The arbitrageurs will see A`s signal when the price is less than \$18. At the same time, based on their information from time 1, their expectation of B`s signal is \$10/2=\$5. The initial stock price at time 2 will be (\$18+ \$5)/2= \$11.5 (see the proof of Lemma 6 in Appendix). The bad news keeps hidden in this case. Investor A will long \$6.5 stock in the derivative market and investor B will short \$8.5 stock in the derivative market, and the implied stock price in the derivative market is \$11.5+(\$6.5-\$8.5) =\$9.5. Since the arbitrageurs have the same confidence in the two markets, the final price in this case will be (\$11.5+\$9.5)/2 =\$10.5, which is equal to the rational price.

At time 3, the real dividend happened, and we assume that it is equal to the mean of A and B`s signal (\$18+\$3)/2 =\$10.5. The whole story is shown in the table below.

Table 1. *A simple case.*

Time	Rational Price	Single Market (Bias with the rational price)	Two Market (Bias with the rational price)
0	\$12.5	\$12.5 (\$0)	\$12.5 (\$0)
1	\$9	\$10 (\$1)	\$9 (\$0)
2	\$10.5	\$11.5 (\$1)	\$10.5 (\$0)
3	\$10.5	\$10.5 (\$0)	\$10.5 (\$0)

According to Table 1, the stock price is overpriced at both time 1 and time 2 because the bad news is hidden in these times. These irrational higher prices result in a stock price crash in time 3. But if there is a perfect derivative market in where the stock can be short or long precisely and without any transaction cost, as long as the arbitrageurs give the derivative market the same trust as the stock market, the final stock price will be the same as the rational price, the price can be displayed quickly and accurately. This decreases the probability of stock crash.

To put it simply, the above story is that if the negative information is released first, the initial pricing given by the stock market is likely to be high due to the short sell constraint on the stock market. What's more, the release of good information after that will further push up share prices, and negative information will continue to be hidden. The accumulation of negative information and the continued overestimation of the stock price will result in a stock price crash after the release of real information.

But if there is a derivative market with no short sell constraint, the stock price will be adjusted in the derivatives market and some signals will be sent to the stock market. Such a mechanism is conducive to better release of information and to make stock prices more reasonable.

The impact of derivatives markets on information flows can be affected in many ways, such as differences in transaction costs between long and short trades, the frictions of information flows between derivatives markets and stock markets. We'll discuss more details in the following section.

3. Model

3.1. A Basic Model

Time 1: the potential for hidden information

At time1, the only private information is held by investor B. Similar to Hong and Stein (2003) [13], there is a cutoff value for S_B such that B`s signal will be hidden when S_B is less than this cutoff value.

Lemma 2. The cut off value for S_B is

$$S_B^* = \frac{2}{3}(V+H) \tag{9}$$

Then for all values of $S_B > S_B^*$, there must be revelation of S_B. The final price will be

$$P_1^F = \begin{cases} \frac{V+H+S_B}{2} + \theta(c_1 - c_2)\frac{(V+H)-S_B}{2}, if\ \frac{2}{3}(V+H) < S_B \le V+H \\ \frac{V+H+S_B}{2} + \theta(c_1 - c_2)\frac{S_B-(V+H)}{2}, if\ V+H < S_B \end{cases} \tag{10}$$

We call these Case 11 ($V+H<S_B$) and Case 12 ($\frac{2}{3}(V+H) < S_B \le V+H$).

Lemma 3. For all values of $S_B \le S_B^*$, S_B will be hidden and the final price will be

$$P_1^F = \frac{2}{3}(V+H) + \theta\left(c_1\left(V+H-\frac{2}{3}(V+H)\right) + c_2\left(S_B - \frac{2}{3}(V+H)\right)\right) \tag{11}$$

We call this Case 2

Case 11 tells us a story that when S_B is "very high" (higher than the ex-ante expectation of A`s signal), B`s signal will be revealed and the bullish investor B will long the stock in both markets while A can only short the stock on the derivative market.

As for Case 12, let`s consider a scenario where the auctioneer start announcing a price $2V+H$, and the arbitrageurs` demand is certain to be negative. Then the auctioneer lowers the price, for any price $V+H>p_1>S_B^*=\frac{2}{3}(V+H)$, investor A will long the stock and the arbitrageurs` demand will be $\mathrm{E}[D|S_B\le p_1]-p_1=\frac{V+H}{2}+\frac{p_1}{4}-p_1=\frac{V+H}{2}-\frac{3p_1}{4}<0$, the auctioneer will keep lowering the price. But since $S_B>S_B^*$, when the auctioneer announcing a price $S_B$, investor B will long the stock and the arbitrageurs see B`s signal and their demand will be $\frac{V+H+S_B}{2}-S_B=\frac{V+H-S_B}{2}>0$, the auctioneer will raise the price until $p_1=\frac{V+H+S_B}{2}$, where the arbitrageurs` demand is zero. This case tells us that when $S_B$ is "moderately high", B`s signal will be revealed. But the stock price is higher than his information, thus B will short the stock at the derivative market.

Case 2 is similar to Case 12 at first, but since $S_B\le S_B^*$, the auctioneer will lower the price until $p_1=S_B^*\ge S_B$ at where the arbitrageurs` demand is zero. In this case, B`s signal will be hidden and the initial stock price is higher than the rational expectation $\frac{V+H+S_B}{2}$. But because B can short the stock market in the derivatives market, the final price, compared to the single market case, may be closer to the rational expectation. For instance, when $c_1=c_2=1,\theta=\frac{1}{2}$, $P_1^F=\frac{2}{3}(V+H)+\frac{1}{2}\left(V+H+S_B-\frac{4}{3}(V+H)\right)=\frac{V+H+S_B}{2}$.

Time 2: previously hidden information may be revealed

The above analysis shows that S_B may not be immediately revealed when S_B is low enough. But at time 2, investor A gets his signal S_A, more information of S_B may come out if S_A is small enough,

Case 1: B`s signal was revealed at time 1. This case is much similar to time 1. S_A will be hidden if it is small enough.

Lemma 4. Assume that S_B has been revealed at time 1. Let the cutoff value for S_A be

$$S_A^*=\frac{2S_B+H}{3} \tag{12}$$

For all values of $S_A>S_A^*$, S_A will be revealed at time 2, and

$$P_2^F=\begin{cases}\frac{S_A+S_B}{2}+\theta(c_1-c_2)\frac{S_A-S_B}{2}, if\ S_B<S_A\\ \frac{S_A+S_B}{2}+\theta(c_1-c_2)\frac{S_B-S_A}{2}, if\ \frac{2S_B+H}{3}<S_A\le S_B\end{cases} \tag{13}$$

We call these Case 1A1 ($S_B<S_A$) and Case 1A2 ($\frac{2S_B+H}{3}<S_A\le S_B$)

Lemma 5. Assume that S_B has been revealed at time 1. For all values of $S_A\le S_A^*$, S_A will be hidden at time 2, and

$$P_2^F=\frac{2S_B+H}{3}+\theta\left(c_2\left(S_A-\frac{2S_B+H}{3}\right)+c_1\left(S_B-\frac{2S_B+H}{3}\right)\right) \tag{14}$$

We call this Case 1B

Case 2: B`s signal was hidden at time 1. This case is more complicated. Intuitively, we believe that the relationship between S_A and S_B affects the release of S_A and S_B. There are three possible results: S_B keep hidden and S_A is revealed; S_A is small enough so that S_B is revealed; both S_A and S_B are revealed.

Lemma 6. Assume that S_B was hidden at time 1. If $S_A>V+H$, then S_A is revealed and S_B continues to be hidden at time 2. The final price is given by

$$P_2^F=\frac{S_A}{2}+\frac{V+H}{6}+\theta\left(c_1\left(S_A-\left(\frac{S_A}{2}+\frac{V+H}{6}\right)\right)+c_2\left(S_B-\left(\frac{S_A}{2}+\frac{V+H}{6}\right)\right)\right) \tag{15}$$

We call this Case 2A

Lemma 7. Assume that S_B was hidden at time 1, and $S_A<V+H$. Let the new cutoff value of S_B

$$S_B^{**}=\frac{2S_A}{3} \tag{16}$$

If $S_B\le S_B^{**}$, then S_A will be revealed and S_B continues to be hidden at time 2. The final price is given by

$$P_2^F=\frac{2S_A}{3}+\theta\left(c_1\left(S_A-\frac{2S_A}{3}\right)+c_2\left(S_B-\frac{2S_A}{3}\right)\right) \tag{17}$$

We call this Case 2B

Lemma 8. Assume that S_B was hidden at time 1, and $S_A<V+H$. As in Lemma 3, the cutoff value of S_A

$$S_A^*=\frac{2S_B+H}{3} \tag{18}$$

If $S_A\le S_A^*$ and $S_B>H$, then S_B will be revealed and S_A will be hidden at time 2. The final price is given by

$$P_2^F=\frac{2S_B+H}{3}+\theta\left(c_2\left(S_A-\frac{2S_B+H}{3}\right)+c_1\left(S_B-\frac{2S_B+H}{3}\right)\right) \tag{19}$$

We call this Case 2C

Lemma 9. Assume that S_B was hidden at time 1. For any values that not already covered in case 1A1, 1A2, 1B, 2A, 2B and 2C. Both S_A and S_B will be revealed and we have

$$P_2^F=\begin{cases}\frac{S_A+S_B}{2}+\theta(c_1-c_2)\frac{S_A-S_B}{2}, if\ S_A>S_B\\ \frac{S_A+S_B}{2}+\theta(c_1-c_2)\frac{S_B-S_A}{2}, if\ S_A\le S_B\end{cases} \tag{20}$$

We call these Case 2D1 ($S_A>S_B$) and Case 2D2 ($S_A>S_B$)

The total story at time 2 is shown in Figure 1

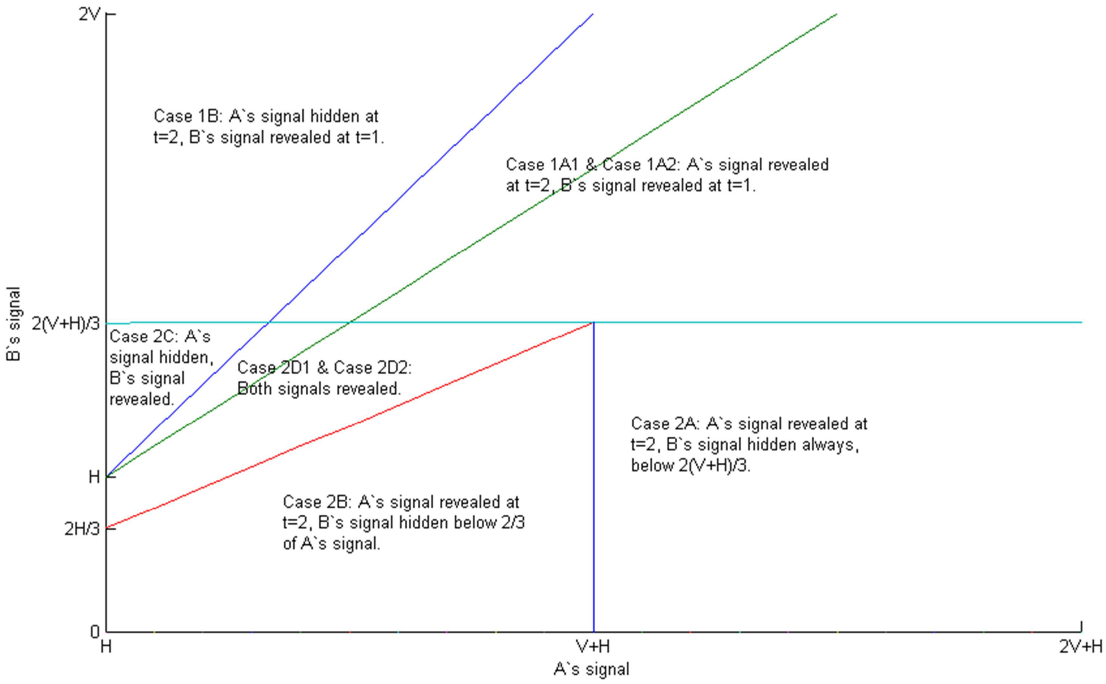

Figure 1. *Partition of equilibrium outcomes at time 2, depending on* S_A *and* S_B.

3.2. General Equilibrium

3.2.1. Extend to Infinite Adjustment

In the previous analysis, we assumed that the derivative market can adjust the stock price only once at each period. Now we extend the single adjustment to the infinite adjustment, that is, for an initial stock price P^S, investor A and B will long and short the stock in the derivative market and the final priceP^F will be a little different from P^S. Then investors will long or short the stock based on the new stock price P^F, and this is the second adjustment of the price. We repeat this adjustment infinitely andthe final result is shown below.

Proposition 1. If the derivatives market can adjust the prices enough. If $\theta \neq 0$, $c_1 + c_2 \neq 0$ and $\theta(c_1 + c_2) \neq 2$, the final price is only about c_1 and c_1 and has nothing to do with θ.

Lemma 10. If the derivatives market can adjust the prices enough. The final price in each case are shown in Table 1.

Table 1. The final price at difference cases, after an infinite adjustment from the derivative market

$$w_1 = \frac{c_1}{c_1 + c_2}, w_2 = 1 - w_1 = \frac{c_2}{c_1 + c_2}$$

Case	Condition	Final price	A`s signal	B`s signal	Story
Time 0					
All values		$V + \frac{H}{2} + (w_1 - w_2)\frac{H}{2}$	No Information	No Information	Initial stock price at time 0
Time 1					
11	$V + H < S_B$	$\frac{V + H + S_B}{2} + (w_1 - w_2)\frac{(V + H) - S_B}{2}$	No Information	Revealed	S_B is big enough and revealed
12	$\frac{2}{3}(V + H) < S_B \leq V + H$	$\frac{V + H + S_B}{2} + (w_1 - w_2)\frac{S_B - (V + H)}{2}$	No Information	Revealed	S_B is relatively big and revealed
2	$S_B \leq \frac{2}{3}(V + H)$	$\frac{2(V + H)}{3} + w_1\left(V + H - \frac{2(V + H)}{3}\right) + w_2\left(S_B - \frac{2(V + H)}{3}\right)$	No Information	Not Revealed	S_B is small and failed to be revealed
Time 2					
1A1	$S_A > \frac{2S_B + H}{3}, S_B > \frac{2(V + H)}{3}, S_A > S_B$	$\frac{S_A + S_B}{2} + (w_1 - w_2)\frac{S_A - S_B}{2}$	Revealed	Revealed	S_B revealed at t=1, S_A is big enough and be revealed at t=2
1A2	$S_A > \frac{2S_B + H}{3}, S_B > \frac{2(V + H)}{3}, S_A \leq S_B$	$\frac{S_A + S_B}{2} + (w_1 - w_2)\frac{S_B - S_A}{2}$	Revealed	Revealed	S_B revealed at t=1, S_A is big enough and be revealed at t=2

1B	$S_A \leq \frac{2S_B+H}{3}, S_B > \frac{2(V+H)}{3}$	$\frac{2S_B+H}{3} + w_2\left(S_A - \frac{2S_B+H}{3}\right) + w_1\left(S_B - \frac{2S_B+H}{3}\right)$	Not Revealed	Revealed	S_B revealed at t=1, S_A is small and be hidden at t=2
2A	$S_A > \mathrm{V+H}, S_B \leq \frac{2(V+H)}{3}$	$\frac{S_A}{2} + \frac{V+H}{6} + w_1\left(S_A - \left(\frac{S_A}{2} + \frac{V+H}{6}\right)\right) + w_2\left(S_B - \left(\frac{S_A}{2} + \frac{V+H}{6}\right)\right)$	Revealed	Not Revealed	S_B is hidden at t=1, S_A is very big so that S_A is revealed at t=2 and S_B keep hidden in t=2
2B	$S_A \leq \mathrm{V+H}, S_B \leq \frac{2S_A}{3}$	$\frac{2S_A}{3} + w_1\left(S_A - \frac{2S_A}{3}\right) + w_2\left(S_B - \frac{2S_A}{3}\right)$	Revealed	Not Revealed	S_B is hidden at t=1, S_A is small but considerably bigger than S_B so that S_A is revealed at t=2 and S_B keep hidden in t=2
2C	$S_A \leq \frac{2S_B+H}{3}, H < S_B \leq \frac{2S_A}{3}$	$\frac{2S_A}{3} + w_2\left(S_A - \frac{2S_A}{3}\right) + w_1\left(S_B - \frac{2S_A}{3}\right)$	Not Revealed	Revealed	S_B is hidden at t=1, S_A is considerably smaller than S_B so that S_B is revealed at t=2 and S_A keep hidden in t=2
2D1	$other\ cases, S_A > S_B$	$\frac{S_A+S_B}{2} + (w_1 - w_2)\frac{S_A - S_B}{2}$	Revealed	Revealed	S_B is hidden at t=1, but the gap between S_A and S_B is not very big so that S_A and S_B are both revealed at t=2
2D2	$other\ cases, S_A \leq S_B$	$\frac{S_A+S_B}{2} + (w_1 - w_2)\frac{S_B - S_A}{2}$	Revealed	Revealed	S_B is hidden at t=1, but the gap between S_A and S_B is not very big so that S_A and S_B are both revealed at t=2
Time 3					
	All values	$\frac{S_A+S_B}{2}$	Revealed	Revealed	Dividend happens, all investors know the real value.

3.2.2. Implications for Return Asymmetries

We define the return at day t as

$$R_t = P_t^F - P_{t-1}^F \tag{21}$$

We use the skewness to measure the asymmetries in the return distribution

$$Skw_t = E[R_t^3], \mathrm{t} = 1,2,3 \tag{22}$$

And we define overall unconditional skewness of short-horizon return as the average skewness over the period

$$Skw_t^S = \frac{\sum_{i=1}^t Skw_i^3}{t} = \frac{\sum_{i=1}^t R_i^3}{t}, \mathrm{t} = 1,2,3 \tag{23}$$

We also define the stock crash as a negative skewness.

Proposition 2. If the derivatives market can adjust the prices enough. When$c_1 = c_2$, the skewness at every day and the unconditional skewness over any periods are 0.

This conclusion is in line with our intuition, that is, a better short-sell environment in the derivative market can effectively promote the release of information. In reality, if we assume that there is no cost to construct a long position in the derivative market ($c_1 = 1$), when the short sell cost is zero ($c_2 = 1$), the exist of derivative will eliminate the stock crash.

Proposition 3. If the derivatives market can adjust the prices enough. If $c_2 < c_1$, no matter how much the differences of opinions (H/2V) is, the skewness at day 1 and day 2 are greater or equal to 0. But there will be a huge negative skewness at day 3 so that the overall skewness turns into negative in many cases after day 3.

This results can be seen in Figure 2 and Figure 3. Given $c_1 = 1$， Figure 2 and Figure 3 show us the relationship between the skewness (daily skewness in Figure 2 and average skewness in Figure 3) and the heterogeneous opinion at different levels of short-sell cost. In Figure 2, we find out that when the short sell cost is great than 0, skewness in time 1 and time 2 is equal or higher than zero, which means an increase in price, but a huge negative skewness appears at time 3 so that the average skewness over 3 days becomes negative in Figure 3, especially when the difference of opinions is higher. Also, the range of variation of skewness increases with the increase in the short sell cost.

This conclusion proves that a derivatives market may reduce the stock crash if the short sell environment is good enough. With the deterioration of the short environment, the derivative may also increase the probability and level of stock crash.

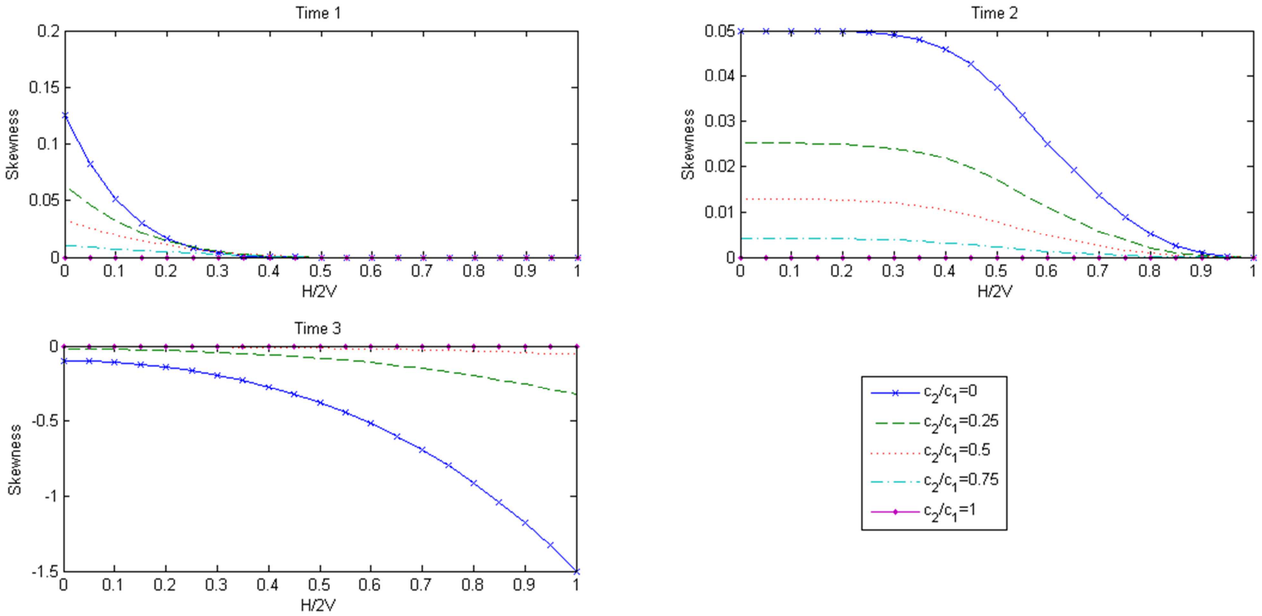

Figure 2. *Daily Skewness and differences of opinions.*

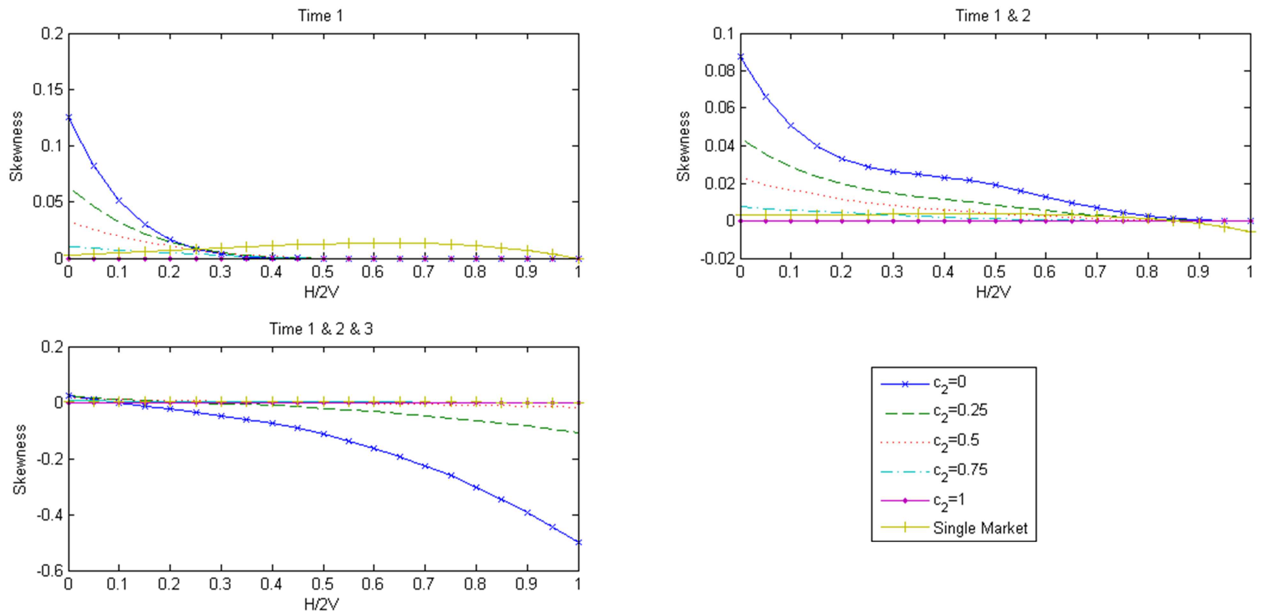

Figure 3. *Overall skewness and differences of opinions.*

Proposition 4. If the derivatives market can adjust the prices enough. The skewness is more negative for a higher level of differences of opinions, but a better short sell environment can reduce the skewness difference.

This result is clearly reflected in Figure 4. In Figure 4, a higher level of opinion difference corresponds to a lower curve. But as the short sell cost decreasing, all the curves are closer to 0, and the distances between these curves are significantly reduced.

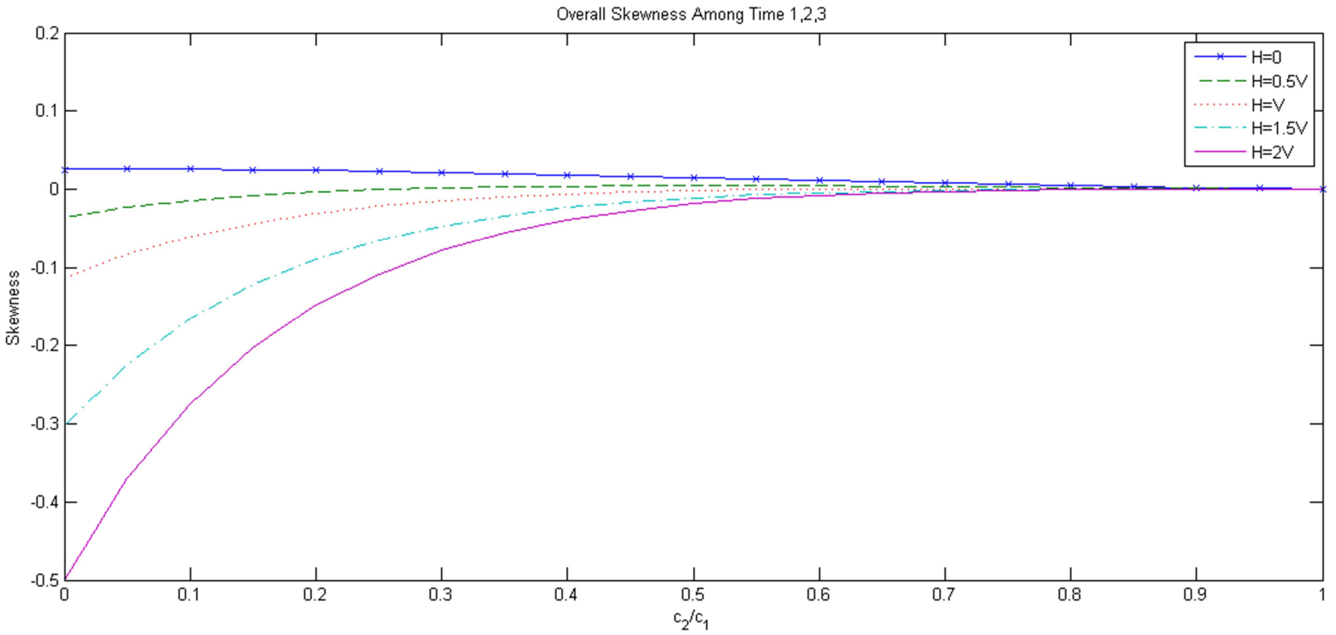

Figure 4. *Overall skewness and short-sell cost.*

4. Conclusion

We construct a four-period-double-market model in this paper. By including the stock market with short selling restrictions and the derivative market without short selling restrictions but with long-short costs in the model, we study the relationship between the asymmetry of long-short cost in derivative market, investors' heterogeneous beliefs and the stock price crash risk.

The asymmetry of short cost in derivatives market will distort the implied price of derivatives market, which will send a wrong message to stock market and intertwined with investors' heterogeneous beliefs in the stock market. Finally, due to the existence of shorting restrictions in the stock market, the risk of stock price risk in the stock market is now different with the single market.

Under the general equilibrium conditions, a derivative market with symmetrical long-short cost can completely eliminate the risk of stock price crash. But if the short-selling cost is higher than the buying cost, the stock price will be overvalued in the early periods, and the negative events will bring more serious stock price crash than the single market situation.

Appendix

Appendix A: Proofs of Lemmas

Proof of Lemma 1. Given the initial stock price P^S and the expected value of investor A and B are P_A and P_B, let $\theta, c_1, c_2 \in [0,1]$, let $P_A > P^S > P_B$. Since $P^S > P_B$, B will not submit an order for the auction price is higher than his expectation.

$$P^F = (1-\theta)P^S + \theta P^O = P^S + \theta\big(c_1(P_A - P^S) + c_2(P_B - P^S)\big) > P_s + \theta c_2(P_B - P^S) \geq P_s + P_B - P^S = P_B \tag{24}$$

Thus, P_B will keep silence after the adjustment.

Proof of Lemma 2. According to Lemma 1, if S_B is hidden at the stock market, it will keep hidden after the adjustment from the derivative market.

In the stock market, since S_B is uniform on $[0,2V]$, if investor B has not submitted an order at trial price p_1, the arbitrageurs` forecast of S_B is

$$\mathrm{E}[S_B | S_B \leq p_1] = \frac{p_1}{2} \tag{25}$$

Hence the risk-neutral arbitrageurs` estimate of the terminal value of the asset is

$$\mathrm{E}[D | S_B \leq p_1] = \frac{V+H}{2} + \frac{p_1}{4} \tag{26}$$

If B keep silence, the auctioneer will lower p_1 until the net demand of the arbitrageur is zero.

$$p_1 = \frac{V+H}{2} + \frac{p_1}{4} \Rightarrow p_1 = \frac{2}{3}(V+H) \tag{27}$$

That is, the lowest price the auctioneer can give is $\frac{2}{3}(V+H)$. So, if $S_B < \frac{2}{3}(V+H)$, B`s information will be hidden. Let $S_B^* = \frac{2}{3}(V+H)$ be the cutoff value of S_B.

When $S_B > S_B^*$, B`s information will be revealed and the initial stock price will be

$$P_1^S = \frac{V+H+S_B}{2} \tag{28}$$

If $V + H > S_B$

$$P_1^F = P_1^S + \theta\big(c_1(V+H-P_1^S) + c_2(S_B - P_1^S)\big) = \frac{V+H+S_B}{2} + \theta(c_1 - c_2)\frac{S_B - (V+H)}{2} \tag{29}$$

If $\frac{2}{3}(V+H) < S_B \le V+H$

$$P_1^F = P_1^S + \theta\big(c_1(S_B - P_1^S) + c_2(V+H-P_1^S)\big) = \frac{V+H+S_B}{2} + \theta(c_1 - c_2)\frac{(V+H)-S_B}{2} \quad (30)$$

Proof of Lemma 3. Since $S_B \le S_B^*$,

$$P_1^S = \mathrm{E}[D|S_B \le S_B^*] = \frac{V+H}{2} + \frac{S_B^*}{4} = \frac{2}{3}(V+H) \quad (31)$$

Also, since $S_B \le S_B^* = \frac{2}{3}(V+H) < V+H$

$$P_1^F = P_1^S + \theta\left(c_1(V+H-P_1^S) + c_2(S_B - P_1^S)\right) = \frac{2}{3}(V+H) + \theta\left(c_1\left(V+H-\frac{2}{3}(V+H)\right) + c_2\left(S_B - \frac{2}{3}(V+H)\right)\right) \quad (32)$$

Proof of Lemma 4. According to Lemma 1, if S_A is hidden at the stock market, it will keep hidden after the adjustment from the derivative market. Since S_A is uniform on $[\mathrm{H}, 2\mathrm{V}+\mathrm{H}]$, if investor A has not submitted an order at trial price p_2, the arbitrageurs` forecast of S_A is

$$\mathrm{E}[S_A|S_A \le p_2] = \frac{H+p_2}{2} \quad (33)$$

Hence the risk-neutral arbitrageurs` estimate of the terminal value of the asset is

$$\mathrm{E}[D|S_A \le p_2] = \frac{H+p_2}{4} + \frac{S_B}{2} \quad (34)$$

If A keep silence, the auctioneer will lower p_2 until the net demand of the arbitrageur is zero.

$$p_2 = \frac{H+p_2}{4} + \frac{S_B}{2} \Rightarrow p_2 = \frac{2S_B+H}{3} \quad (35)$$

That is, the lowest price the auctioneer can give is $\frac{2S_B+H}{3}$. So, if $S_A < \frac{2S_B+H}{3}$, A`s information will be hidden. Let $S_A^* = \frac{2S_B+H}{3}$ be the cutoff value of S_A.

When $S_A > S_A^*$, A`s information will be revealed and the initial stock price will be

$$P_2^S = \frac{S_A+S_B}{2} \quad (36)$$

If $S_A > S_B$

$$P_2^F = P_2^S + \theta\big(c_1(S_A - P_2^S) + c_2(S_B - P_2^S)\big) = \frac{S_A+S_B}{2} + \theta(c_1 - c_2)\frac{S_A-S_B}{2} \quad (37)$$

If $\frac{2S_B+H}{3} < S_A \le S_B$

$$P_2^F = P_2^S + \theta\big(c_1(S_B - P_2^S) + c_2(S_A - P_2^S)\big) = \frac{S_A+S_B}{2} + \theta(c_1 - c_2)\frac{S_B-S_A}{2} \quad (38)$$

Proof of Lemma 5. Since $S_A \le S_A^*$,

$$P_2^S = \mathrm{E}[D|S_A \le S_A^*] = \frac{S_B}{2} + \frac{S_A^*+H}{4} = \frac{2S_B+H}{3} \quad (39)$$

Also, since S_B was revealed, $S_B > S_B^* = \frac{2}{3}(V+H)$

$$S_B - \frac{2S_B+H}{3} = \frac{S_B-H}{3} > \frac{2(V+H)-3H}{9} = \frac{2V-H}{9} \ge 0 \Rightarrow S_B > \frac{2S_B+H}{3} \ge S_A \quad (40)$$

Thus, the final price will be

$$P_2^F = P_2^S + \theta\big(c_1(S_A - P_2^S) + c_2(S_B - P_2^S)\big) = \frac{2S_B+H}{3} + \theta\left(c_2\left(S_A - \frac{2S_B+H}{3}\right) + c_1\left(S_B - \frac{2S_B+H}{3}\right)\right) \quad (41)$$

Proof of Lemma 6. Since $S_B \le S_B^* = \frac{2}{3}(V+H)$ and S_B was hidden at time 1. S_A will be revealed if $S_A > \frac{2}{3}(V+H)$ and the initial stock price will be

$$P_2^S = \frac{S_A}{2} + \frac{S_B^*}{4} \quad (42)$$

If $S_A > V+H$

$$P_2^S = \frac{S_A}{2} + \frac{S_B^*}{4} > \frac{\mathrm{V+H}}{2} + \frac{S_B^*}{4} = \frac{2}{3}(\mathrm{V+H}) = S_B^* \ge S_B \quad (43)$$

Thus, S_B will keep hidden in this case. Since $S_B \le P_2^S < S_A$ and the final price will be

$$P_2^F = P_2^S + \theta\big(c_1(S_A - P_2^S) + c_2(S_B - P_2^S)\big) = \frac{S_A}{2} + \frac{V+H}{6} + \theta\left(c_1\left(S_A - \left(\frac{S_A}{2} + \frac{V+H}{6}\right)\right) + c_2\left(S_B - \left(\frac{S_A}{2} + \frac{V+H}{6}\right)\right)\right) \quad (44)$$

Proof of Lemma 7. When S_A is revealed first, when S_B is hidden at a price p_2

$$\mathrm{E}[S_B | S_B \le p_2] = \frac{p_2}{2} \quad (45)$$

The risk-neutral arbitrageurs` estimate of the terminal value of the asset is

$$\mathrm{E}[D | S_B \le p_2] = \frac{S_A}{2} + \frac{p_2}{4} \quad (46)$$

If B keep silence, the auctioneer will lower p_2 until the net demand of the arbitrageur is zero.

$$p_2 = \frac{S_A}{2} + \frac{p_2}{4} \Rightarrow p_2 = \frac{2}{3} S_A \quad (47)$$

That is, the lowest price the auctioneer can give is $\frac{2}{3}(V+H)$. So, if $S_B < \frac{2}{3} S_A$, B`s information will be hidden. Let $S_B^{**} = \frac{2}{3} S_A$ be the new cutoff value of S_B.

If $S_B \le S_B^{**}$, then S_A will be revealed and S_B continues to be hidden at time 2. The initial stock price will be

$$P_2^S = \frac{S_A}{2} + \frac{S_B^{**}}{4} = \frac{2S_A}{3} \quad (48)$$

It is easy to see that $S_B < P_2^S < S_A$ and the final price is

$$P_2^F = P_2^S + \theta\big(c_1(S_A - P_2^S) + c_2(S_B - P_2^S)\big) = \frac{2S_A}{3} + \theta\left(c_1\left(S_A - \frac{2S_A}{3}\right) + c_2\left(S_B - \frac{2S_A}{3}\right)\right) \quad (49)$$

Proof of Lemma 8. When $\frac{2}{3}(V+H) \ge S_B > H$ and $S_A \le S_A^* = \frac{2S_B+H}{3} < \frac{2S_B+S_B}{3} = S_B$, S_B will be revealed before S_A. If S_A is hidden at a price p_2

$$\mathrm{E}[S_A | S_A \le p_2] = \frac{H+p_2}{2} \quad (50)$$

The risk-neutral arbitrageurs` estimate of the terminal value of the asset is

$$\mathrm{E}[D | S_A \le p_2] = \frac{H+p_2}{4} + \frac{S_B}{2} \quad (51)$$

If A keep silence, the auctioneer will lower p_2 until the net demand of the arbitrageur is zero.

$$p_2 = \frac{H+p_2}{4} + \frac{S_B}{2} \Rightarrow p_2 = \frac{2S_B+H}{3} \quad (52)$$

That is, the lowest price the auctioneer can give is $\frac{2S_B+H}{3}$. So, if $S_A < S_A^* = \frac{2S_B+H}{3}$, A`s information will be hidden. The initial stock price will be

$$P_2^S = \mathrm{E}[D | S_A \le S_A^*] = \frac{S_B}{2} + \frac{S_A^*+H}{4} = \frac{2S_B+H}{3} \quad (53)$$

It is easy to see that $S_A < P_2^S < S_B$, and the final price will be

$$P_2^F = P_2^S + \theta\big(c_2(S_A - P_2^S) + c_1(S_B - P_2^S)\big) = \frac{2S_A}{3} + \theta\left(c_2\left(S_A - \frac{2S_B+H}{3}\right) + c_1\left(S_B - \frac{2S_B+H}{3}\right)\right) \quad (54)$$

Proof of Lemma 9. For any other cases that are not covered by Lemma 2 to Lemma 8, both S_B will be revealed at time 1 and S_A will be revealed at time 2. According to Figure 1, when $S_B \in [\frac{2}{3}H, \frac{2}{3}(V+H)]$ and $S_A \in [\frac{2S_B+H}{3}, \frac{3}{2}S_B]$ both signals will be revealed. This is one case that when S_B is not too small and S_A is not too far away from S_B.

The initial stock price will be

$$P_2^S = \frac{S_A+S_B}{2} \quad (55)$$

If $S_A > S_B$

$$P_2^F = P_2^S + \theta\big(c_1(S_A - P_2^S) + c_2(S_B - P_2^S)\big) = \frac{S_A+S_B}{2} + \theta(c_1 - c_2)\frac{S_A - S_B}{2} \quad (56)$$

If $S_A \le S_B$

$$P_2^F = P_2^S + \theta\big(c_2(S_A - P_2^S) + c_1(S_B - P_2^S)\big) = \frac{S_A+S_B}{2} + \theta(c_1 - c_2)\frac{S_B - S_A}{2} \quad (57)$$

Proof of Lemma 10. Based on Proposition 1, if the derivatives market can adjust the prices enough

$$P^F = P^S + \frac{c_1}{c_1+c_2}(P_A - P^S) + \frac{c_2}{c_1+c_2}(P_B - P^S) \quad (58)$$

Where P^S is the initial stock market price, P_A and P_B are the expected value of long side and short side investor, $w_1 = \frac{c_1}{c_1+c_2}$ and $w_2 = \frac{c_2}{c_1+c_2} = 1 - w_1$.

Compare to the case where the derivatives market can adjust the prices only once.

$$P^F = P^S + \theta(c_1(P_A - P^S) + c_2(P_B - P^S)) \quad (59)$$

We can treat that the general case as a special case when $\theta = \frac{1}{c_1+c_2}$. Then we can get all results in Table 1.

Appendix B: Proofs of Propositions

Proof of Proposition 1. Given the initial stock price P^S and the expected value of investor A and B are P_A and P_B, let $\theta, c_1, c_2 \in [0,1]$, let $P_A > P^S > P_B$, let P_t^F be the final price after the t-th adjustment by the derivative market.

Case 1. If $c_1 = c_2 = 0$ or $\theta = 0$

$$P_t^F \equiv P^S$$

$$P^F = \lim_{t\to\infty} P_t^F = P^S \tag{60}$$

Case 2. If $c_1 + c_2 \neq 0$ and $\theta \neq 0$

$$P_0^F = P^S$$

$$P_1^F = P_0^F + \theta\big(c_1(P_A - P_0^F) + c_2(P_B - P_0^F)\big) \tag{61}$$

According to Lemma 1, if the bad news was hidden at the initial stock price, it will keep hidden after the adjustment from the derivative market. Thus, the stock price will change from P^S to P_1^F. Then investors long or short the stock in the derivative market after they observe the new stock price P_1^F

$$P_2^F = P_1^F + \theta\big(c_1(P_A - P_1^F) + c_2(P_B - P_1^F)\big) \tag{62}$$

After this process of adjustment has been repeated t-times, the new final price is

$$P_t^F = P_{t-1}^F + \theta\Big(c_1(P_A - P_{t-1}^F) + c_2(P_B - P_{t-1}^F)\Big) = P_{t-1}^F\big(1 - \theta(c_1 + c_2)\big) + \theta(c_1P_A + c_2P_B)$$

$$P_t^F - \frac{c_1P_A + c_2P_B}{c_1 + c_2} = \big(1 - \theta(c_1 + c_2)\big)\left(P_{t-1}^F - \frac{c_1P_A + c_2P_B}{c_1 + c_2}\right)$$

$$P_t^F - \frac{c_1P_A + c_2P_B}{c_1 + c_2} = \big(1 - \theta(c_1 + c_2)\big)^t\left(P_0^F - \frac{c_1P_A + c_2P_B}{c_1 + c_2}\right)$$

$$P_t^F = \frac{c_1P_A + c_2P_B}{c_1 + c_2} + \big(1 - \theta(c_1 + c_2)\big)^t\left(P_0^F - \frac{c_1P_A + c_2P_B}{c_1 + c_2}\right) \tag{63}$$

Case 2.1 If $\theta(c_1 + c_2) = 2$, that is, $\theta = c_1 = c_2 = 1$

$$P_t^F = \begin{cases} P^S, t = 2k \\ P_A + P_B - P^S, t = 2k + 1 \end{cases}, k = 1,2,3 \ldots \tag{64}$$

Case 2.2 If $\theta(c_1 + c_2) \neq 2$

$$1 - \theta(c_1 + c_2) \in (-1,1)$$

$$\lim_{t\to\infty}\big(1 - \theta(c_1 + c_2)\big)^t = 0$$

$$P^F = \lim_{t\to\infty} P_t^F = \frac{c_1P_A + c_2P_B}{c_1 + c_2} \tag{65}$$

Thus, if $\theta \neq 0$, $c_1 + c_2 \neq 0$ and $\theta(c_1 + c_2) \neq 2$, the final price

$$P^F = \frac{c_1P_A + c_2P_B}{c_1 + c_2} = P^S + \frac{c_1}{c_1 + c_2}(P_A - P^S) + \frac{c_2}{c_1 + c_2}(P_B - P^S) \tag{66}$$

is only about c_1 and c_1 and has nothing to do with θ. In fact, θ determines the rate at which the price converges to the final price.

Proof of Proposition 2.

Given the following price at every day.

$$P_0^F = V + \frac{H}{2} + (w_1 - w_2)\frac{H}{2}$$

$$P_{1,Case1.1}^F = \frac{V + H + S_B}{2} + (w_1 - w_2)\frac{(V + H) - S_B}{2}$$

$$P_{1,Case1.2}^F = \frac{V + H + S_B}{2} + (w_1 - w_2)\frac{S_B - (V + H)}{2}$$

$$P_{1,Case2}^F = \frac{2(V + H)}{3} + w_1\left(V + H - \frac{2(V + H)}{3}\right) + w_2\left(S_B - \frac{2(V + H)}{3}\right)$$

$$P_{2,Case1A1}^F = \frac{S_A + S_B}{2} + (w_1 - w_2)\frac{S_A - S_B}{2}$$

$$P_{2,Case1A2}^F = \frac{S_A + S_B}{2} + (w_1 - w_2)\frac{S_B - S_A}{2}$$

$$P_{2,Case1B}^F = \frac{2S_B + H}{3} + w_2\left(S_A - \frac{2S_B + H}{3}\right) + w_1\left(S_B - \frac{2S_B + H}{3}\right)$$

$$P^F_{2,Case2A} = \frac{S_A}{2} + \frac{V+H}{6} + w_1\left(S_A - \left(\frac{S_A}{2} + \frac{V+H}{6}\right)\right) + w_2\left(S_B - \left(\frac{S_A}{2} + \frac{V+H}{6}\right)\right)$$

$$P^F_{2,Case2B} = \frac{2S_A}{3} + w_1\left(S_A - \frac{2S_A}{3}\right) + w_2\left(S_B - \frac{2S_A}{3}\right)$$

$$P^F_{2,Case2C} = \frac{2S_A}{3} + w_2\left(S_A - \frac{2S_A}{3}\right) + w_1\left(S_B - \frac{2S_A}{3}\right)$$

$$P^F_{2,Case2D1} = \frac{S_A + S_B}{2} + (w_1 - w_2)\frac{S_A - S_B}{2}$$

$$P^F_{2,Case2D2} = \frac{S_A + S_B}{2} + (w_1 - w_2)\frac{S_B - S_A}{2}$$

$$P^F_3 = \frac{S_A + S_B}{2}$$

At day 1:

$$\mathrm{E}[R_1^3] = \mathrm{E}[R_1^3|S_B > V+H]\Pr(S_B > V+H) + \mathrm{E}\left[R_1^3 \middle| \tfrac{2}{3}(V+H) < S_B \le V+H\right]\Pr(\tfrac{2}{3}(V+H) < S_B \le V+H) + \mathrm{E}\left[R_1^3 \middle| S_B \le \tfrac{2}{3}(V+H)\right]\Pr(S_B \le \tfrac{2}{3}(V+H)) \quad (67)$$

Since S_B is uniform on $[0,2V]$
When $H \le V, V+H \le 2V$

$$\mathrm{E}[R_1^3] = \frac{1}{2V}\left[\int_{V+H}^{2V}\left(P^F_{1,Case1.1} - P^F_0\right)^3 dy + \int_{\frac{2}{3}(V+H)}^{V+H}\left(P^F_{1,Case1.2} - P^F_0\right)^3 dy + \int_0^{\frac{2}{3}(V+H)}\left(P^F_{1,Case2} - P^F_0\right)^3 dy\right] \quad (68)$$

When $H > V, V+H > 2V$

$$\mathrm{E}[R_1^3] = \frac{1}{2V}\left[\int_{\frac{2}{3}(V+H)}^{2V}\left(P^F_{1,Case1.2} - P^F_0\right)^3 dy + \int_0^{\frac{2}{3}(V+H)}\left(P^F_{1,Case2} - P^F_0\right)^3 dy\right] \quad (69)$$

Define $c_2 = k_c c_1$ and $H = 2k_H V$. The skewness at day 1 is

$$Skw_1 = \mathrm{E}[R_1^3] = \begin{cases} -V^3(2k_H - 1)^2(k_c - 1)\frac{4k_H^2(3k_c^2 - 3k_c + 1) + 4k_H(k_c^2 + k_c - 1) + k_c^2 + k_c + 1}{8(k_c+1)^3}, k_H \in [0,0.5) \\ 0, k_H \in [0.5,1] \end{cases} \quad (70)$$

At day 2:
When $H \le V, V+H \le 2V$
$\mathrm{EE}[R_2^3] = [R_2^3|\text{Case1.1}]\Pr(\text{Case1.1}) + \mathrm{E}[R_2^3|\text{Case1.2}]\Pr(\text{Case1.2}) + \mathrm{E}[R_3^3|\text{Case2}]\Pr(\text{Case2}) = \mathrm{E}[R_2^3|\text{Case1.1}]\frac{V-H}{2V} + \mathrm{E}[R_2^3|\text{Case1.2}]\frac{V+H}{6V} + \mathrm{E}[R_3^3|\text{Case2}]\frac{V+H}{3V}$ (A.46)
Here

$$\mathrm{E}[R_2^3|\text{Case1.1}] = \frac{1}{2V-(V+H)}\frac{1}{2V}\int_{V+H}^{2V}\left[\int_y^{2V+H}\left(P^F_{2,Case1A1} - P^F_{1,Case1.1}\right)^3 dx + \int_{\frac{2y+H}{3}}^{y}\left(P^F_{2,Case1A2} - P^F_{1,Case1.1}\right)^3 dx + \int_H^{\frac{2y+H}{3}}\left(P^F_{2,Case1B} - P^F_{1,Case1.1}\right)^3 dx\right] dy;$$

$$\mathrm{E}[R_2^3|\text{Case1.2}] = \frac{1}{V+H-\frac{2}{3}(V+H)}\frac{1}{2V}\int_{\frac{2}{3}(V+H)}^{V+H}\left[\int_y^{2V+H}\left(P^F_{2,Case1A1} - P^F_{1,Case1.2}\right)^3 dx + \int_{\frac{2y+H}{3}}^{y}\left(P^F_{2,Case1A2} - P^F_{1,Case1.2}\right)^3 dx + \int_H^{\frac{2y+H}{3}}\left(P^F_{2,Case1B} - P^F_{1,Case1.2}\right)^3 dx\right] dy;$$

$$\mathrm{E}[R_2^3|\text{Case2}] = \frac{1}{\frac{2}{3}(V+H)}\frac{1}{2V}\sum_{i=1}^3 E_i \quad \text{where}$$

$$E_1 = \int_0^{\frac{2H}{3}}\left[\int_{V+H}^{2V+H}\left(P^F_{2,Case2A} - P^F_{1,Case2}\right)^3 dx + \int_H^{V+H}\left(P^F_{2,Case2B} - P^F_{1,Case2}\right)^3 dx\right] dy \ ;$$

$$E_2 = \int_{\frac{2H}{3}}^{H}\left[\int_{V+H}^{2V+H}\left(P_{2,Case2A}^F - P_{1,Case2}^F\right)^3 dx + \int_{\frac{3y}{2}}^{V+H}\left(P_{2,Case2B}^F - P_{1,Case2}^F\right)^3 dx + \int_{H}^{\frac{3y}{2}}\left(P_{2,Case2D1}^F - P_{1,Case2}^F\right)^3 dx\right] dy \ ;$$

$$E_3 = \int_{H}^{\frac{2}{3}(V+H)}\left[\int_{V+H}^{2V+H}\left(P_{2,Case2A}^F - P_{1,Case2}^F\right)^3 dx + \int_{\frac{3y}{2}}^{V+H}\left(P_{2,Case2B}^F - P_{1,Case2}^F\right)^3 dx + \int_{y}^{\frac{3y}{2}}\left(P_{2,Case2D1}^F - P_{1,Case2}^F\right)^3 dx + \int_{\frac{2y+H}{3}}^{\frac{3y}{2}}\left(P_{2,Case2D2}^F - P_{1,Case2}^F\right)^3 dx + \int_{H}^{\frac{2y+H}{3}}\left(P_{2,Case2C}^F - P_{1,Case2}^F\right)^3 dx\right] dy \ ;$$

When $H > V, V + H > 2V$

$$E[R_2^3] = E[R_{2.2}^3|\text{Case1.2}]\Pr(\text{Case1.2}) + E[R_1^3|\text{Case2}]\Pr(\text{Case2}) = E[R_{2.2}^3|\text{Case1.2}]\frac{2V-H}{3V} + E[R_2^3|\text{Case2}]\frac{V+H}{3V} \tag{71}$$

Here

$$E[R_{2.2}^3|\text{Case1.2}] = \frac{1}{2V-\frac{2}{3}(V+H)}\frac{1}{2V}\int_{\frac{2}{3}(V+H)}^{2V}\left[\int_{y}^{2V+H}\left(P_{2,Case1A1}^F - P_{1,Case1.2}^F\right)^3 dx + \int_{\frac{2y+H}{3}}^{y}\left(P_{2,Case1A2}^F - P_{1,Case1.2}^F\right)^3 dx + \int_{H}^{\frac{2y+H}{3}}\left(P_{2,Case1B}^F - P_{1,Case1.2}^F\right)^3 dx\right] dy$$

Define $c_2 = k_c c_1$ and $H = 2k_H V$. The skewness at day 2 is

$$Skw_2 = E[R_2^3] = \begin{cases} -V^3(k_c-1)\frac{8k_H^5(-3k_c^2+3k_c-1)+20k_H^4(2k_c^2-k_c)-20k_H^3k_c^2+k_c^2+k_c+1}{20(k_c+1)^3}, k_H \in [0,0.5) \\ V^3(k_H-1)^3(k_c-1)\frac{2k_H^2(k_c^2-3k_c+3)-k_H(4k_c^2-7k_c+2)+2k_c^2-k_c+1}{5(k_c+1)^3}, k_H \in [0.5,1] \end{cases} \tag{72}$$

At day 3:

$$E[R_3^3] = E[R_3^3|\text{Case1.1}\&Case1.2]\Pr(\text{Case1.1}\&Case1.2) + E[R_3^3|\text{Case2}]\Pr(\text{Case2}) = E[R_3^3|\text{Case1.1}\&Case1.2]\frac{2V-H}{3V} + E[R_3^3|\text{Case2}]\frac{V+H}{3V} \tag{73}$$

Here

$$E\left[R_3^3\big||\text{Case1.1}\&Case1.2\right] = \frac{1}{2V-\frac{2}{3}(V+H)}\frac{1}{2V}\int_{\frac{2}{3}(V+H)}^{2V}\left[\int_{y}^{2V+H}(P_3^F - P_{2,Case1A1}^F)^3 dx + \int_{\frac{2y+H}{3}}^{y}(P_3^F - P_{2,Case1A2}^F)^3 dx + \int_{H}^{\frac{2y+H}{3}}(P_3^F - P_{2,Case1B}^F)^3 dx\right] dy;$$

$$E[R_3^3|\text{Case2}] = \frac{1}{\frac{2}{3}(V+H)}\frac{1}{2V}\sum_{i=1}^{3} E_i;$$

where

$$E_1 = \int_{0}^{\frac{2H}{3}}\left[\int_{V+H}^{2V+H}(P_3^F - P_{2,Case2A}^F)^3 dx + \int_{H}^{V+H}(P_3^F - P_{2,Case2B}^F)^3 dx\right] dy \ ;$$

$$E_2 = \int_{\frac{2H}{3}}^{H}\left[\int_{V+H}^{2V+H}(P_3^F - P_{2,Case2A}^F)^3 dx + \int_{\frac{3y}{2}}^{V+H}(P_3^F - P_{2,Case2B}^F)^3 dx + \int_{H}^{\frac{3y}{2}}(P_3^F - P_{2,Case2D1}^F)^3 dx\right] dy \ ;$$

$$E_3 = \int_{H}^{\frac{2}{3}(V+H)}\left[\int_{V+H}^{2V+H}(P_3^F - P_{2,Case2A}^F)^3 dx + \int_{\frac{3y}{2}}^{V+H}(P_3^F - P_{2,Case2B}^F)^3 dx + \int_{y}^{\frac{3y}{2}}(P_3^F - P_{2,Case2D1}^F)^3 dx + \int_{\frac{2y+H}{3}}^{\frac{3y}{2}}(P_3^F - P_{2,Case2D2}^F)^3 dx + \int_{H}^{\frac{2y+H}{3}}(P_3^F - P_{2,Case2C}^F)^3 dx\right] dy;$$

Define $c_2 = k_c c_1$ and $H = 2k_H V$. The skewness at day 3 is

$$Skw_3 = V^3(k_c-1)^3\frac{-k_H^5+5k_H^4+10k_H^2+1}{10(k_c+1)^3} \tag{74}$$

It is easy to see that when $k_c = 1$, we have $Skw_1 = Skw_2 = Skw_3 = 0$ and the unconditional skewness over any periods $Skw_t^S = \frac{\sum_{i=1}^{t} Skw_i^3}{t} = 0$ for t = 1,2,3

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