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Analysis and Development of Price Models in the System MEANS of Intersectoral Balance

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Abstract: At present, monetary models of equilibrium prices have received a very wide and varied application. In this paper it is proved that the application of the currently existing monetary models of the intersectoral balance is associated with significant methodical errors. It is shown that price models based on the input balance provides such a balance only in unreal cases when the prices of all sectors are the same. For realistic conditions, when these requirements are not met, errors for these models reach unacceptable values. This article proposes and thoroughly investigated new price models based on output balances which do not have the said drawbacks. It is mathematically proven that the new models satisfy the output balances and do not have methodical errors. When used, they provide zero output imbalances for both theoretical and realistic data packets. Also in the paper the calculation results are presented both for the existing monetary models of input-output balance and for new price models using a wide set of initial data. The calculations performed confirm the theoretical conclusions of the article. It is also proved that the Leontief price model is a special case of the generalized model of price indices proposed in the work.

Keywords: Price Model, Leontief Model, Generalized Price Indices Model, Input Balance, Output Balance, Error

1. Introduction

Price models in the theory of intersectoral balance in mathematical and applied implications are a unique phenomenon in the environment of systems analysis models. Mathematically, these models are underdetermined systems of algebraic equations, the dimension of which is twice less than the number of unknowns. A researcher in the conditions of task is given the possibility (more precisely, requirement) to redefine the algebraic system founded specific features of problem. Otherwise, the model provides an infinite number of solutions.

More importantly, price models, which have long been widely used in Input-Output tools, have been put into practice without strict mathematical justification. Prolonged unsuccessful attempts of the author to build a mathematically reasonable grounds for the adequacy of existing monetary price models led him to the conclusion that they contain methodical errors. But, despite the fact that the author managed to prove that this is indeed the case (will be further), the monetary models of equilibrium prices in combination with the models of Leontief (1953) and Ghosh (1958) have received a very wide and diverse application (especially recently) [1, 2].

There is a significant attention in recent years publications to the direct impact of the price factor on the functioning of both national and interregional economics. Here are some typical examples. For example, Flaen A. et al. found that since the beginning of 2018, after an unprecedented increase in tariffs in the US manufacturing sector contrary to expectations a reduction in employment, rising producer prices and production resources costs take place [3].

As well using Leontief price model Mukaramah Harun et al. showed that the abolition of fuel subsidies in Malaysia's manufacturing sectors has led to an average increase in producer prices of 32%, which has a serious impact on inflation in the country [4]. Sharp changes in world oil prices could jeopardize Malaysia's economic stability.

Recently, an important theoretical and practical result has been obtained to take into account the impact of price changes on the matrix of direct (intersectoral) costs, wherein it is proposed to adjust the matrix coefficients by using Cobb-

Douglas functions or elasticity coefficients [5].

Theoretically and practically important research were carried out by Wiebe K. S using a closed Input-Output model, which analyzes the planet's economy as a whole [6]. Compared to the usual scenario, this model shows that world production of materials is reduced by about 10% and the impact on employment is small but positive. Wherein the transition from the resource sector to the service sector will provide more opportunities for highly qualified professionals and women.

Deviations between direct prices based on labor costs, production and market prices according to the Input-Output tables in China in the period from 1990 to 2012 were studied in the research of Minqi Li [7]. It was found that cross-deviations between direct and market prices average 17 - 18 percent. Variations in direct prices can explain about 70 percent of changes in market prices.

Bhattacharyya A. made a comparison of market and shadow prices in the US economics and social sphere using the means of intersectoral balance [8]. This research allows to identify areas in which it is necessary to change the combination of results or costs to improve social welfare.

Based on the means of intersectoral balance and tools of integrated network analysis, Chen B. researched global energy flows in international trade, taking into account the vulnerability of the environment at the global, regional and national levels [9]. In this study at a global level the nature of the small world is revealed, in which economices are closely interconnected through the transfer of energy. It is shown that at the national level, key countries (USA, China, Germany) are at the forefront of network centralization, and the security of the implemented energy supply is assessed for each economy.

A study of the daily economic costs of COVID-19 mitigation control to inform the governments of Brazil and Colombia was carried out by Haddad E. A. in a short time frame using a cross-sectoral balance methodology and tools [10].

Yu K. D. S. carried out an important study to model the economic impact of COVID-19 on the economies of affected countries [11]. A roadmap has been developed to assess the vulnerability of the supply chain through disease outbreaks at the company level, national and global levels.

Increased attention in recent publications is paid to the study using the means of intersectoral balance of interaction and interconnection in the economic and environmental spheres of certain groups of countries, their sectors and regions.

In particular, population growth and climate change have made food, energy and water security a global issue. To address this issue, Tabatabaie S. M. H developed a problem-oriented Input-Output model, which provides a food-energy-water (FEW) relationship for the Northwest Pacific [12]. The results showed that agricultural crops have the highest sensitivity to water and energy consumption. To minimize costs and environmental impact, more use should be made of surface water, hydroelectric power and wind energy.

Wang X.-C carried out research on the relationship between water, energetics and carbon emissions in China on the basis of intersectoral balance tools [13]. It was found that the results of the interaction between water, energy and carbon emissions in light, heavy industry and services were comparable, agriculture accounted for about 64% of the country's water supply. It is also shown that indicators of water and energy consumptions and carbon emissions can significantly affect the country's sustainable development strategies.

Due to a number of objective factors, Iran-Iraq and Turkey have joined forces to address the strategic security issue of Water-Energy-Food (WEF). According to the UN, the demand for water, energy and food in these countries has grown significantly over the past 20 years. This can exacerbate the conflict probability, especially across transboundary water resources. The interrelationships of the WEF as a holistic approach to finding regional solutions to common problems in these countries have been studied by Zarei M. [14]. Cooperation and interaction between the scientific community and decision-makers are vital to the complex challenges of WEF security management and development.

White D. J. used the transnational interregional input-output approach to analyze the relationship between East Asia WEF (Japan, China, and South Korea) to assess competing this resource needs and environmental performance [15]. This analysis demonstrates the hidden virtual flows of water, energy and food embodied in intra-regional and transnational trade. China has been shown to be a purely virtual exporter of WEF resources due to its trade in low value-added and high-pollution sectors.

It turns out that the Input-Output methodology can be effectively applied even for the analysis of such complex processes as Brexit. Giammetti R. carried out the research what provides an opportunity to identify areas that are key in the structure of relations between the United Kingdom and the European Union [16]. It is possible to assess which tariffs are the most influential in the negotiations, which export sector needs to be intensified, and which imports should be protected. The results show that Brexit will be a problem not only for Britain, but any form of it can affect the global production system.

The possibilities of the Input-Output toolkit are used quite effectively by Mandras G. in the analysis of trade integration of the economies of the Western Balkans [17]. Their results allows to identify industries associated with high economic effects and to form an idea of sectoral interdependence of economics. The multi-regional data set was used to study the international integration of the region's economics in order to participate in global value chains. Shown, that although this indicator has recently tended to grow, some economies benefit from this is more than others.

The use of methodology and means of intersectoral balance often deals with the action of counter-orientation factors. In Brazil, in particular, greenhouse gas (GHG) emissions have been reduced by 12% over the last three decades due to reduced deforestation. At the same time, GHG emissions without this factor increased by 18%, and gross domestic product increased by 17%. As GHG emission reduction activities are quite costly, there is a problem of ensuring sustainable development of the country. A comparison of GHG emission multipliers in the Brazilian economics with employment and income multipliers (especially in agriculture) provided an opportunity for de Santana Ribeiro L. C. to develop appropriate solutions to this problem [18].

The analysis shows that the use of structures, capabilities and means of intersectoral balance over time significantly expands geographically and increases in quantitative and qualitative terms. Characteristically, these positive phenomena occur despite the existence (as mentioned above) of certain methodical errors in the key models of the Input-Output (IO) apparatus, namely, in the equilibrium monetary price models. We can be sure that the removal of these errors will contribute to even greater dissemination and increase the quality of research results provided by the methodology of intersectoral balance.

The purpose of this publication is mathematical and computational confirmation of the fact of methodical errors that occur in existing monetary price models (IO), identifying their causes, development, description and research of new IO models for this purpose and the possibilities of their application.

2. Analysis of Existing Price Models of Intersectoral Balance

The methodological basis for building equilibrium price models in the Input-Output system is a set of IO matrices shown in (1).

$$\begin{array}{c} \\ 1 \\ i \\ j \\ n \end{array} \overset{\begin{array}{cccc} 1 & i & X\;j & n \end{array}}{\begin{bmatrix} x_{11} & x_{1i} & x_{1j} & x_{1n} \\ x_{i1} & x_{ii} & x_{ij} & x_{in} \\ x_{j1} & x_{ji} & x_{jj} & x_{jn} \\ x_{n1} & x_{ni} & x_{nj} & x_{nn} \end{bmatrix}}; \overset{f}{\begin{bmatrix} f_1 \\ f_i \\ f_j \\ f_n \end{bmatrix}}; \overset{x}{\begin{bmatrix} x_1 \\ x_i \\ x_j \\ x_n \end{bmatrix}}; \tag{1}$$

$$\begin{bmatrix} v_1 & v_i & v_j & v_n \end{bmatrix} v;$$

$$\begin{bmatrix} z_1 & z_i & z_j & z_n \end{bmatrix} z = x'.$$

Here $i, j = \overline{1, n}$ – sector numbering, x_{ij} – elements of the matrix of intermediate sales X; f, x, v, z – vectors of final demand, output, value added and total input, respectively. In balanced table 1 are always provided, as is known, dependencies:

$$z_i = x_i, \tag{2}$$

$$\sum_{j=1}^{n} v_j = \sum_{i=1}^{n} f_i, \tag{3}$$

as well as the balance of output

$$\sum_{j=1}^{n} x_{ij} + f_i = x_i \tag{4}$$

and input balance

$$\sum_{i=1}^{n} x_{ij} + v_j = x_j. \tag{5}$$

We draw attention to the fundamental need to ensure the dependences (2) - (5) in the construction of correct Input-Output models.

Currently, in the theory and practice of intersectoral balance, a significant number of price models have been developed, which are naturally divided into two groups, namely, models whose matrices are formed on physical data, and monetary models. As points out by de Mesnard L. physical models are usually used only in theoretical research, because the formation of their matrices is associated with difficulties in providing statistical information [19].

The available price models of the intersectoral balance use value added in one form or another as initial information. Therefore, these models are based on the input balance (5). The most popular of the available schemes for the formation of price models is given in the publication Handbook of Input-Output Table Compilation and Analysis, UN (1999) [20]. It is used as a basis the input balance (5). In this case, each of its j-th column is divided into output $\bar{x}_j$, $j = \overline{1-n}$ in units of output and get a system of equations

$$p = (I - A')^{-1} \gamma, \tag{6}$$

where p, γ – price vectors, parts of value added per unit of output

$$\gamma_j = v_j / \bar{x}_j, \tag{7}$$

$A = \left[a_{ij}\right] = \left[x_{ij} / x_j\right]$ – matrix from the Leontief output model for its (model) monetary form. In the econometric literature, system (6), (7) is called the "Dual model".

Developers and users of model (6), (7) consider it a monetary model of equilibrium prices in the system of models of intersectoral balance. The author [21] and a number of other authors in earlier publications formulate this statement clearly and unambiguously. We will show that this statement is not always true. First, we show that the model (6), (7) in the general case does not satisfy the input balance equation (5). To do this, consider model (6), (7) in an expanded form

$$p_j - \frac{x_{1j}}{x_j} p_1 - \frac{x_{ij}}{x_j} p_i - \frac{x_{jj}}{x_j} p_j - \frac{x_{nj}}{x_j} p_n = \frac{v_j}{x_j}, \quad j = \overline{1,n}, \tag{8}$$

and taking into account the obvious dependencies

$$x_j = p_j \bar{x}_j, \quad j = \overline{1,n}. \tag{9}$$

$$x_{ji} = p_i \bar{x}_{ji}, \quad i,j = \overline{1,n} \tag{10}$$

we get the expression for the input balance

$$x_j = x_{1j} \frac{p_1}{p_j} + x_{ij} \frac{p_i}{p_j} + x_{jj} + x_{nj} \frac{p_n}{p_j} + v_j, \tag{11}$$

or

$$x_j = \sum_{i=1}^{n} x_{ij} \frac{p_i}{p_j} + v_j, \quad j = \overline{1,n}. \tag{12}$$

Dependence (12) gives grounds to conclude that model (6), (7) is not a model from the class of Input-Output models, because it does not satisfy the input balance (5) in the general case. This balance is satisfied according to (12) only in one degenerate case, namely when prices in all sectors have the same value

$$p_i = p_j, \quad i,j = \overline{1,n}. \tag{13}$$

It is clear that dual model cannot have any practical application, because in real calculations case (13) (equality of prices in all sectors) is nonsense. A very limited consideration of this model is found in purely theoretical analysis, although not always with positive results, as will be discussed below.

We show further that the scheme of obtaining the price model described in Handbook of Input-Output Table Compilation and Analysis, (1999), Eurostat Manual of Supple, Use and Input-Output Tables, (2008) and Handbook on Supply and Use Tables and Input Output-Tables with Extensions and Applications, (2018), does not lead to the model (6), (7), but to a completely different result [20, 24, 25]. To do this, first, using the input balance (5) in monetary form and the dependence (10), we obtain a system

$$\begin{aligned}
&p_1 \bar{x}_{11} + p_i \bar{x}_{i1} + p_j \bar{x}_{j1} + p_n \bar{x}_{n1} + v_1 = p_1 \bar{x}_1, \\
&p_1 \bar{x}_{1i} + p_i \bar{x}_{ii} + p_j \bar{x}_{ji} + p_n \bar{x}_{ni} + v_i = p_i \bar{x}_i, \\
&p_1 \bar{x}_{1j} + p_i \bar{x}_{ij} + p_j \bar{x}_{jj} + p_n \bar{x}_{nj} + v_j = p_j \bar{x}_j, \\
&p_1 \bar{x}_{1n} + p_i \bar{x}_{in} + p_j \bar{x}_{jn} + p_n \bar{x}_{nn} + v_n = p_n \bar{x}_n.
\end{aligned} \tag{14}$$

Dividing each of the equations of system (14) by the corresponding $\bar{x}_j$, we obtain the final system in an expanded form

$$\begin{aligned}
&p_1 \frac{\bar{x}_{11}}{\bar{x}_1} + p_i \frac{\bar{x}_{i1}}{\bar{x}_1} + p_j \frac{\bar{x}_{j1}}{\bar{x}_1} + p_n \frac{\bar{x}_{n1}}{\bar{x}_1} + \frac{v_1}{\bar{x}_1} = p_1, \\
&p_1 \frac{\bar{x}_{1i}}{\bar{x}_i} + p_i \frac{\bar{x}_{ii}}{\bar{x}_i} + p_j \frac{\bar{x}_{ji}}{\bar{x}_i} + p_n \frac{\bar{x}_{ni}}{\bar{x}_i} + \frac{v_i}{\bar{x}_i} = p_i, \\
&p_1 \frac{\bar{x}_{1j}}{\bar{x}_j} + p_i \frac{\bar{x}_{ij}}{\bar{x}_j} + p_j \frac{\bar{x}_{jj}}{\bar{x}_j} + p_n \frac{\bar{x}_{nj}}{\bar{x}_j} + \frac{v_j}{\bar{x}_j} = p_j, \\
&p_1 \frac{\bar{x}_{1n}}{\bar{x}_n} + p_i \frac{\bar{x}_{in}}{\bar{x}_n} + p_j \frac{\bar{x}_{jn}}{\bar{x}_n} + p_n \frac{\bar{x}_{nn}}{\bar{x}_n} + \frac{v_n}{\bar{x}_n} = p_n,
\end{aligned} \tag{15}$$

which is obviously presented in matrix form

$$(I - \bar{A}')\, p = \gamma, \quad \gamma_j = v_j / \bar{x}_j, \quad j = \overline{1,n}, \tag{16}$$

where the matrix

$$\bar{A} = \left[\bar{a}_{ij}\right]\left[\bar{x}_{ij} / \bar{x}_j\right], \quad i,j = \overline{1,n} \tag{17}$$

is a matrix of coefficients of intermediate sales in physical form.

Thus, model (6), (7) cannot satisfy the input balance (5), because (as can be seen from (14)) it is satisfied by a completely different model (16), (17).

Although outwardly the models (6), (7) and (16), (17) are similar, their essences differ radically due to the striking discrepancy, as is known, of the values of the elements of the matrices A and $\bar{A}$. Note that the dependences (14) - (16) are obtained by identical transformations, i.e., model (16), (17) does not contain methodical errors.

Thus, scheme of transformation of the monetary model IO into the price model (6), (7) what is cited in Handbook of Input-Output Table Compilation and Analysis, (1999), Eurostat Manual of Supple, Use and Input-Output Tables, (2008) and Handbook on Supply and Use Tables and Input Output-Tables with Extensions and Applications, (2018) cannot be carried out, because it leads to the price model in physical form (16) [20, 24, 25].

As shown by the calculations of equilibrium prices on examples with real monetary data (Appendices **A1**, **A6**), the methodical errors of model (6), (7) can reach such values that they can not be ignored.

To ensure the possibility of working with input data in monetary form, Leontief developed a model of price indices [22]. This model is cited, in particular, by de Mesnard L., Miller R. E., Eurostat Manual of Supple, Use and Input-Output Tables, (2008), Handbook on Supply and Use Tables and Input Output-Tables with Extensions and Applications, (2018) [19, 23-25]. The essence of the model of price indices is that its two states are considered - basic and current. This model differs from model (6), (7) only by the right part (7), namely, it (model) has the form

$$(I - A')\beta = v_c, \tag{18}$$

where β_j – the price index of the j-th sector,

$$v_{cj} = v_j / x_j, \quad j = \overline{1,n}, \tag{19}$$

and besides for the base state

$$v_{cj}^0 = v_j^0 / x_j^0, \quad j = \overline{1,n}, \tag{20}$$

and for the current state

$$v_{cj} = v_j / x_j^0, \quad j = \overline{1,n}. \tag{21}$$

By direct substitution of (22) into (18), one can verify that for the base state the price modules $\left|p_j^0\right|$ satisfy the input balance (5).

$$\left|p_j^0\right| = 1, \quad j = \overline{1,n}. \tag{22}$$

Therefore, the solution of system (18) directly provides the vector of price indices β_j, $j = \overline{1,n}$ of the current state of the model in relation to its base state.

This model of price indices has a very important drawback. It does not have (ignored) the output in its physical units, which sharply narrows the application scope of this model. In addition, ignoring its value often leads (shown below) to unacceptably large errors.

De March et al. proposed a generalized price model (given in Eurostat Manual of Supple, Use and Input-Output Tables (2008)) in the form of

$$p = (I - A')^{-1} qv, \tag{23}$$

which at $A = [a_{ij}] = \left[x_{ij} / x_j\right]$ and $[qv]_j = v_{cj}$ according to (21) becomes a model of equilibrium price indices (18), and when $\bar{A} = [\bar{a}_{ij}] = \left[\bar{x}_{ij} / \bar{x}_j\right]$ and $[qv]_j = v_j / \bar{x}_j$ – turns into a price model (16) [24]. Model (23) is used unchanged in Handbook on Supply and Use Tables and Input Output-Tables with Extensions and Applications, UN (2018) [25]. The use of model (23) does not provide additional advantages or disadvantages compared to models (16) and (18).

The validity of the conclusions regarding the model of price indices (18) - (21) is actually confirmed by de Mesnard L. [19].

Thus, the analysis of present monetary price models of the intersectoral balance gives grounds to claim that they all have certain defects and need to be clarified or developed.

3. New Monetary Price Models in the Input-Output System

This section presents two new monetary price models, namely, strictly speaking the price model and the model of price indices that do not contain methodical errors. Unlike the models discussed in the previous section, these models are not based on the input balance (5), but on the output balance (4). To build such a model, we first use the balance (4) in expanded form and, dividing each of its equations by the corresponding output in physical form $\bar{x}_i$, $i = \overline{1,n}$, we obtain a system of equations (24).

$$\begin{aligned}
&\frac{x_{11}}{\bar{x}_1} + \frac{x_{1i}}{\bar{x}_1} + \frac{x_{1j}}{\bar{x}_1} + \frac{x_{1n}}{\bar{x}_1} + \frac{f_1}{\bar{x}_1} = \frac{x_1}{\bar{x}_1} = p_1, \\
&\frac{x_{i1}}{\bar{x}_i} + \frac{x_{ii}}{\bar{x}_i} + \frac{x_{ij}}{\bar{x}_i} + \frac{x_{in}}{\bar{x}_i} + \frac{f_i}{\bar{x}_i} = \frac{x_i}{\bar{x}_i} = p_i, \\
&\frac{x_{j1}}{\bar{x}_j} + \frac{x_{ji}}{\bar{x}_j} + \frac{x_{jj}}{\bar{x}_j} + \frac{x_{jn}}{\bar{x}_j} + \frac{f_j}{\bar{x}_j} = \frac{x_j}{\bar{x}_j} = p_j, \\
&\frac{x_{n1}}{\bar{x}_n} + \frac{x_{ni}}{\bar{x}_n} + \frac{x_{nj}}{\bar{x}_n} + \frac{x_{nn}}{\bar{x}_n} + \frac{f_n}{\bar{x}_n} = \frac{x_n}{\bar{x}_n} = p_n.
\end{aligned} \tag{24}$$

Using the dependence (9), the system of equations (24) is transformed into the system (25).

$$\begin{aligned}
&\frac{x_{11}}{x_1} p_1 + \frac{x_{1i}}{x_1} p_1 + \frac{x_{1j}}{x_1} p_1 + \frac{x_{1n}}{x_1} p_1 + \frac{f_1}{x_1} = p_1, \\
&\frac{x_{i1}}{x_i} p_i + \frac{x_{ii}}{x_i} p_i + \frac{x_{ij}}{x_i} p_i + \frac{x_{in}}{x_i} p_i + \frac{f_i}{x_i} = p_i, \\
&\frac{x_{j1}}{x_j} p_j + \frac{x_{ji}}{x_j} p_j + \frac{x_{jj}}{x_j} p_j + \frac{x_{jn}}{x_j} p_j + \frac{f_j}{x_j} = p_j, \\
&\frac{x_{n1}}{x_n} p_n + \frac{x_{ni}}{x_n} p_n + \frac{x_{nj}}{x_n} p_n + \frac{x_{nn}}{x_n} p_n + \frac{f_n}{x_n} = p_n.
\end{aligned} \tag{25}$$

The system of equations (25) is a price monetary model in an expanded form. After entering the notation

$$s_{ii} = \sum_{j=1}^{n} x_{ij} / x_i, \quad i = \overline{1,n} \tag{26}$$

we obtain a monetary price model in matrix form

$$(I - S)p = \mu, \tag{27}$$

in which the matrix S has a diagonal structure with non-zero elements (26), p – price vector, μ – vector particles of final demand μ_i per physical unit of output of the i-th sector

$$\mu_i = f_i / \bar{x}_i, \quad i = \overline{1,n}. \tag{28}$$

To verify the conformity of model (27) to the structure of IO, its i-th equation

$$p_i - \left(\sum_{j=1}^{n} x_{ij}\right) p_i = f_i / \bar{x}_i, \quad i = \overline{1,n}$$

is sufficient multiply by $\bar{x}_i$, as a result, it is converted into the equation of the balance of output (4). This indicates that the new model has no methodical errors.

The main destination of the new model is to determine equilibrium prices and solve related problems based on the Input-Output methodology without methodical errors and limitations. No less important possibilities of its application are connected with the fact that the matrix (***I*** - ***S***) in the system (27) has a diagonal structure. This feature allows to find solutions of this system in analytical form. In turn, this provides an opportunity to determine the equilibrium prices and their changes in the current state through their value in the base state. That is, the prospect of building a new model of price indices opens up without the limitations and shortcomings that have, in particular, models (16), (17) and (18).

Using the output balance (4), we obtain the diagonal element of the matrix ***S*** in the form

$$s_{ii} = \left(x_i - f_i\right) / x_i \tag{29}$$

After that, the analytical solution of the system (27) is determined as

$$p_i = \mu_i x_i / f_i, \quad i = \overline{1,n}. \tag{30}$$

The presence of dependence (30) makes it possible to determine price indices in monetary form.

For the current state, the analytical solution of the system of equations (27) has the form

$$p_i = \mu_i x_i / f_i, \quad i = \overline{1,n} \tag{31}$$

and for the base state

$$p_i^0 = \mu_i^0 x_i^0 / f_i^0, \quad i = \overline{1,n}. \tag{32}$$

Determination of price indices β_i, $i = \overline{1,n}$ carried out as the ratio of their value p_i in the current state to the value p_i^0 in the base state

$$\beta_i = p_i / p_i^0 = \mu_i x_i f_i^0 / \mu_i^0 x_i^0 f_i, \quad i = \overline{1,n}. \tag{33}$$

With the involvement of (28) and after the introduction of symbols $\Delta x_i = x_i - x_i^0$ and $\Delta \bar{x}_i = \bar{x}_i - \bar{x}_i^0$ we obtain the final dependence for price indices

$$\beta_i = (1 + \Delta x_i / x_i^0) / (1 + \Delta \bar{x}_i / \bar{x}_i^0), \quad i = \overline{1,n}. \tag{34}$$

The generalized model of price indices also satisfies the balance of output (4). To prove this, we use the *i*-th equation of system (27) in the form

$$\beta_i p_i^0 - \left(\sum_{j=1}^{n} x_{ij} / x_i\right) \beta_i p_i^0 = f_i / \bar{x}_i,$$

or

$$p_i - \left(\sum_{j=1}^{n} x_{ij} / x_i\right) p_i = f_i / \bar{x}_i,$$

or

$$x_i - \sum_{j=1}^{n} x_{ij} = f_i.$$

The last expression is the modified balance of output (4) in monetary form. Thus, the generalized model of price indices also does not contain methodical errors.

Model (34) is called a generalized model of price indices, because when $\Delta \bar{x}_i = 0$ it becomes Leontief price model. This can be seen, in particular, by comparing the prices of Appendix A4 at $\Delta \bar{x}_i = 0$ and prices of Appendix A2. That is, the Leontief price model (18) - (21) is a special case of model (34).

When performing both theoretical and applied research, it is advisable to have not only dependencies for price indices, but also expressions for the deviation (change) of equilibrium prices compared to the baseline. Such expressions are obtained, in particular, by forming dependencies

$$\left(p_i - p_i^0\right) / p_i^0 = \beta_i - 1, \tag{35}$$

or

$$\Delta p_i = \left(\beta_i - 1\right) p_i^0, \quad i = \overline{1,n}. \tag{36}$$

Formula (36) is needed, in particular, to determine price multipliers.

It is worth emphasizing that the dependence (30) provides another proof that the model (27) has no methodical errors. It is enough for this to substitute (28) in (30), after which we obtain an obvious dependence $p_i = x_i / \bar{x}_i$, $i = \overline{1,n}$.

In contrast to the Leontief price model, the generalized model of price indices allows to use as a base state of the Input-Output system its arbitrary state, in which only the appropriate conditions of balance are provided.

4. Examples

The capabilities and indicators of the above price models were demonstrated by calculations in which the initial data were formed on the basis of information provided in Eurostat Manual of Supple, Use and Input-Output Tables (2008)

(Table 1) [24]. This is a system of real reporting data in the Input-Output format (Germany, 1995).

Table 1. *Reporting system Input-Output.*

		Agricul-ture	Manufac-turing	Construc-tion	Trade	Business services	Other services	Millions of Euro Final demand	Output
		1	2	3	4	5	6	7	8
		Intermediate sales Z						f	x
1	Agriculture	1131	25480	1	607	710	762	15219	43910
2	Manufacturing	7930	304584	64167	41082	11981	30360	619342	1079446
3	Construction	426	7334	3875	5296	23457	9155	196063	245606
4	Trade	3559	72717	14190	74399	10835	21008	343355	540063
5	Business services	3637	96115	31027	65755	193176	34223	268554	692487
6	Other services	1552	14986	1747	11225	15058	22070	442280	508918
Value added		25675	558230	130599	341699	437270	391340	v	
Input, total		43910	1079446	245606	540063	692487	508918	$z = x'$	

In order to study as widely as possible the properties and capabilities of the studied models on the basis of information of table 1 and other additional sources, two universal input data packets were formed. The first of them (theoretical) contains input data, which as a result of their application give the values of equilibrium prices in the district of the unit. The second packet (realistic) provides data that cause the resulting sectoral equilibrium prices to differ several times depending on the technological nature of the sector, which is close to reality.

In this paper, the equilibrium prices were calculated according to the four models discussed above.

4.1. Equilibrium Price Model in the Input-Output System, Built on the Basis of Input Balance

$$p=(I-A')^{-1}\gamma,\ \gamma_j=v_j/\bar{x}_j,\ j=\overline{1-n}$$

(Dual Model)

This model requires the availability of output indicators in physical units, which are not in Eurostat Manual of Supple, Use and Input-Output Tables, (2008) [24]. Therefore, the condition of equality according to the modulus of monetary and physical indicators of output in the base state was used for calculations (Table 1)

$$\left|x_j^0\right|=\left|\bar{x}_j^0\right|,\ j=\overline{1,n}, \qquad (37)$$

which analytically provides equality (22).

These features in no way limit the content of the conclusions in the comparative analysis. Deviations of initial values v_j and $\bar{x}_j$ are chosen small, but such (± 30%) that the deviation of equilibrium prices was noticeable. All data and results of calculations of equilibrium prices on this model according to a theoretical package are provided in Appendix A1. Equilibrium prices locate in the district of the unit.

4.2. Leontief Price Model

$$(I-A')\beta=v_c,\ p_j=\beta_j p_j^0,\ j=\overline{1,n}$$

When calculating prices for this model, the base state was considered to be the state shown in table 1. The interrelation between monetary and physical output was determined by the dependence (37), so there was an equality (22). This led, in turn, that the equilibrium prices for this model modulo coincide with price indices

$$\left|p_j\right|=\beta_j,\ j=\overline{1,n}.$$

The initial data according to the theoretical packet and indicators of price indices and equilibrium prices according to the specified model are given in Appendix A2. It is noteworthy that the equilibrium prices for the considered models (items 1 to 2) differ significantly.

4.3. Equilibrium Price Model Based on the Output Balance

$$(I-S)\,p=\mu,\ \mu_i=f_i/\bar{x}_i,\ i=\overline{1,n}$$

Price calculations for this model were performed on the basis of the table 1 and taking into account the dependences (37), (22) for the possibility of further comparison of the obtained indicators with the indicators of other models. Unlike the models according to items 1 to 2, this model allows to check the execution of the output balance (4) in monetary form. In addition, this model does not use value added v as the initial vector, but final demand f. Because in the calculations according to items 1 to 2 used a vector v that differs from the vector v^0 according to table 1, to ensure compliance with the requirements of the balance of the system of tables IO (1) it is necessary to change the vector f. This was done according to the dependence

$$f=(I-A)(I-B')^{-1}v,$$

because $f=(I-A)x$ and $x=(I-B')^{-1}v$, where B – Ghosh matrix. The calculation of equilibrium prices according to this model and verification of the balance of output with their use are given in Appendix A3.

4.4. Generalized Model of Price Indices and Equilibrium Prices

$$\beta_i=(1+\Delta x_i/x_i^0)/(1+\Delta\bar{x}_i/\bar{x}_i^0),\ p_i=\beta_i p_i^0,\ i=\overline{1,n}$$

In the calculations for this model were also used the data of table 1 and dependences (37), (22). Calculations of price indices and equilibrium prices for this model are provided in

Appendix A4. It is noteworthy that the values of prices for this model and the model based on the output balance (4) (Appendix A3) are completely the same. It is very important that the equilibrium prices obtained for these models exactly satisfy the balance of output according to the theoretical data packet.

Significant results were obtained by digital modeling of these models using a realistic data package. Combined with similar indicators for equilibrium prices in the theoretical packet, this provides an opportunity to carry out an extended comparative analysis of the results. A comparative analysis of the results for the theoretical packet is provided in Appendix A5 and for the realistic packet in Appendix A6.

5. Analysis of Results

Theoretically, all known price models, built on the basis of input balance, are empirical, developed without a strict mathematical justification and have methodical errors. In particular, model (6), (7), the right part of which uses the share of value added per physical unit of output (7), can be considered a model of the class Input-Output only if prices in all sectors are the same. Otherwise, this model according to (12) does not provide a input balance (5), which is an integral requirement for IO class models. In practical application this model will provide a methodical error, because in real calculations price equality in all sectors is impossible.

The model of price indices (Leontief price model) (18) - (21) is forced to use as its base state the initial data, which are even less realistic than in model (6), (7). If in model (6), (7) methodical errors will be absent at equality of prices in all sectors, then to achieve such result in model of price indices in its basic state the prices in all sectors should be not only identical, but also equal units. The use of such a basic state of the model narrows the scope of its possible applications to unrealistic options.

Another very important disadvantage of this price indices model is that the equations of its current state do not take into account the output in units of production, despite the fact that this indicator has a decisive influence as on prices as on their indices. Therefore, with a significant (but realistic) difference between the modules $\left|x_j\right|$ and $\left|\bar{x}_j\right|$ price errors p_j according to this model can reach unacceptable values.

This paper proposes and investigates in detail two monetary price models, namely, the price model based on the output balance and the price indices model formed on the basis of this price model.

It is theoretically proved that both the new price model and the generalized model of price indices satisfy the balance of output (4), i.e., they do not have methodical errors. Both series of calculations carried out in the course of research (Appendix A1 - A6) confirm and concretize the theoretical conclusions and provisions given above. Appendices A1 - A4 show the equilibrium prices for the respective models, and Appendices A5, A6 - their errors for the theoretical and realistic packages, respectively. For comparative analysis, the equilibrium price indicators obtained using the price models (26) - (28) and (34) - (36) were chosen as a reference for both data packages, as they satisfy the balances of output in monetary form. In addition, these models provide a complete match of the significatives of the obtained equilibrium prices, which also confirms their accuracy (Appendices A3 - A6). Deviations of input data at value added in the calculations according to the theoretical packet were chosen in the range of ± 30% in all Appendices A1 - A6. The same deviations for this packet were synchronously selected for physical output in Appendices A1 - A5. These deviations for value added are quite realistic, but the deviations of physical output in real cases can be much larger. In particular, for high-tech sectors (industry) monetary output in modulus may be several times higher than the physical, which is not typical, for example, for agriculture.

To bring the equilibrium prices closer to reality, a analogous series of calculations was performed according to the realistic data packet (Appendix A6). In this packet, the ratio of sectoral monetary output to physical was in the range of 1.5 -5.9 depending on the technological characteristics of the sectors.

A comparative analysis of the obtained results of determining the equilibrium prices in the intersectoral balance on four mathematical models and two packages of initial data allows us to make the following generalizations.

All existing mathematical models of equilibrium prices contain in their structure certain methodical errors, which depend on the nature of the input data. In particular, for Leontief price model, these errors are absent when prices in all sectors of the module are equal to one. The price model, built on the input balance, does not provide errors only if the modulo prices in all sectors are equal. However, these conditions in the practical application of these models can not be met. Even in the conditions of application of the theoretical data packet, when the obtained equilibrium prices for these models are close to one (Appendices A1, A2), the error rate for the model according to the input balance is 18.24% and for Leontief price model - 30% (Appendix A5). When calculating a realistic data packet, these models provide catastrophic errors, namely, the vector of errors for the model according to the input balance has a norm $\left\|\delta p_j\right\|$ = 116,8% and according to Leontief price model – $\left\|\delta p_j\right\|$ = 83,2% (Appendix A6). If the results of the application of the theoretical data packet these two models should be assessed as grossly inaccurate, then the results of the use of a realistic data packet should be classified as incorrect. The accuracy of the two new price models, namely, the model based on the output balance and the generalized model of price indices was checked by calculating the imbalance of output

$$\delta x_i = x_i - \sum_{j=1}^{n} p_i \bar{x}_{ij} - f_i. \tag{38}$$

According to Appendices A5 and A6, the imbalance (38) is zero both when calculating prices for the theoretical data packet and when using a realistic packet for this purpose. It is also important that the eponymous indicators of price obtained using these models coincide in five to six decimal places.

6. Conclusions

Theoretical research and digital experiments have shown that the currently known monetary price models of the intersectoral balance theory do not contain methodical errors only in some cases that are degenerate. In particular, Leontief price model has no methodical errors only in the case when all sectoral equilibrium prices are modulo one. The monetary price model, built on the input balance, will not have methodical errors, provided that the equilibrium prices obtained according to it will all be the same. Failure to comply with these requirements leads to a violation of the monetary input balance, and, as a consequence, to the emergence of methodical errors in decisions. By digital modeling, arrays of methodical errors are obtained when using these models depending on the input information. When using the theoretical data packet (changes in value added and physical outputs are in the range of ± 30%, the resulting prices are placed near the unit) the error rate for Leontiev price model reaches 30% and for the model formed on the input balance - more than 18%. Both of these significatives are unacceptable for practical use.

When a realistic data packet was used (the ratio of monetary and physical output modules increased several times), the error rates for these models increased catastrophically: for the Leontief price model it was about 83% and for the model formed on the basis of input balance - almost 117%. According to such error rates, the relevant models should be classified as incorrect.

Therefore, the currently existing price models and models derived from them analyzed in the paper are not suitable for practical use according to realistic data, as such use leads to unacceptably large errors. They can be used only in situations where the equilibrium sectoral prices of the module are equal to each other, or (moreover) equal to one. That is, they can be used only in theoretical research as degenerate cases.

Proposed and comprehensively studied in the work of two new price models (price model, built on the basis of output balance, and the corresponding model of price indices) are devoid of these shortcomings. It is mathematically proved that both new models satisfy the balances of output. The consequence of this is that when they are used in digital modeling, zero output imbalances are provided for both theoretical and realistic data packets. In contrast to the Leontief price model, the generalized model of price indices allows us to use as the base state of the Input-Output system its arbitrary state, in which only the appropriate conditions of balance are provided.

It is proved that the Leontief price model is a special case of the generalized model of price indices proposed in the work.

Appendix

Appendix A1. Calculation of Equilibrium Prices in the Input-Output System Based on the Input Balance

$$(I-A')^{-1}p=\gamma,\ \gamma_j=v_j/\bar{x}_j,\ j=\overline{1,n}$$

Output in units of output $\bar{x}_j\left(\left|\bar{x}_j^0\right|=\left|x_j^0\right|\right)$

Sector	1	2	3	4	5	6
$\bar{x}_j$	30737	1403280	171924	702082	484741	661593
% $\bar{x}_j^0$	70	130	70	130	70	130

Added value v_j, v_j^0 – data table 1

Sector	1	2	3	4	5	6
v_j	17972	725699	91419	444209	306089	508742
% v_j^0	70	130	70	130	70	130

Value added per unit of output $\gamma_j=v_j/\bar{x}_j$

Sector	1	2	3	4	5	6
v_j	0,5847	0,517145	0,53174	0,632702	0,631449	0,768965

Equilibrium prices $p=(I-A')^{-1}\gamma$

Sector	1	2	3	4	5	6
p_j	1,0055	1,00509	1,274254	1,00562	1,01334	1,00666

Appendix A2. Calculation Equilibrium Prices According to the Leontief Price Model $(I-A')\beta = v_c,\ p_j = \beta_j\, p_j^0,\ \beta$ ***– Vector of Price Indices,*** $j=\overline{1,n}$

Added value v_j ; v_j^0 – data table 1

Sector	1	2	3	4	5	6
v_j	17972	725699	91419	444209	306089	508918
% v_j^0	70	130	70	130	70	130

Output $x=(I-B')^{-1}v$, B – Ghosh matrix

Sector	1	2	3	4	5	6
x_j	37801	1298739	214502	651677	517841	632283

Value added per unit of monetary output $v_{cj}=v_j / x_j^0,\ x_j^0$ – data table 1

Sector	1	2	3	4	5	6
v_{cj}	0,409292	0,672288	0,372218	0,822513	0,442014	0,999654

Price indices $\beta=(I-A')^{-1}\ v_c$

Sector	1	2	3	4	5	6
β_j	0,86085	1,20322	0,87334	1,20668	0,7478	1,24241

Prices according to the Leontief model of price indices $p_j=\beta_j p_j^0$

Sector	1	2	3	4	5	6
p_j	0,86085	1,20322	0,87334	1,20668	0,7478	1,24241

Appendix A3. Calculation of Equilibrium Prices in the Input-Output System According to the Model Developed on the Basis of the Output Balance $(I-S)\,p=\mu,\ \mu_i=f_i/\overline{x}_i$

Output in units of output $\overline{x}_i\left(\left|\overline{x}_i^0\right|=\left|x_i^0\right|\right)$

Sector	1	2	3	4	5	6
$\overline{x}_i$	30737	1403280	171924	702082	484741	661593
% $\overline{x}_i^0$	70	130	70	130	70	130

Added value v_j, v_j^0 – data table 1

Sector	1	2	3	4	5	6
v_j	17972	725699	91419	444209	306089	508742
% v_j^0	70	130	70	130	70	130

Final demand $f=(I-A)(I-B')^{-1}\ v$, B – Ghosh matrix

Sector	1	2	3	4	5	6
f_i	3960	773159	166621	424753	105651	559165

% f_i^0	26	125	85	124	39	126

Output $x = (I - A)^{-1} f$

Sector	1	2	3	4	5	6
x_i	37801	1298739	214502	651677	517840	632283

Equilibrium prices $p = (I - S)^{-1} \mu$

Sector	1	2	3	4	5	6
p_i	1,22982	0,9255	1,24766	0,92821	1,06828	0,9557

Imbalances of outputs $\delta x_i = x_i - p_i \overline{x}_i$

Sector	1	2	3	4	5	6
x_i	37801	1298739	214502	651677	517841	632283
$p_i \overline{x}_i$	37800	1298736	214502	651679	517839	632284
δx_i	0	0	0	0	0	0

Appendix A4. Calculation of Price Indices and Equilibrium Prices According to the Generalized Model

$$\beta_i = \left(1 + \Delta x_i / x_i^0\right) / \left(1 + \Delta \overline{x}_i / \overline{x}_i^0\right), \; p_i = \beta_i p_i^0, \; i = \overline{1,n}$$

Initial data (theoretical packet)

Monetary output in the basic state x_i^0 (table 1)

Sector	1	2	3	4	5	6
x_i^0	43910	1079446	245606	540063	692487	508918

Physical output in the basic state $\left|\overline{x}_i^0\right| = \left|x_i^0\right|, \; i = \overline{1,n}$

Monetary output in its current state, Appendix A3

Sector	1	2	3	4	5	6
x_i	37801	1298739	214502	651677	517841	632283

Physical output in the current state, Appendix A3

Sector	1	2	3	4	5	6
$\overline{x}_i$	30737	1403280	171924	702082	484741	661593

Estimated indicators, price indices

Sector	1	2	3	4	5	6
$\Delta x_i = x_i - x_i^0$	-6109	219293	-31104	111614	-174646	123365
$\Delta \overline{x}_i = \overline{x}_i - \overline{x}_i^0$	-13173	323834	-73682	162019	-207746	152675
$1 + \Delta x_i / x_i^0$	0,86087	1,20315	0,87336	1,20667	0,7478	1,24241
$1 + \Delta \overline{x}_i / \overline{x}_i^0$	0, 7	1,3	0,7	1,3	0,7	1,3
β_i	1,22982	0,9255	1,24765	0,92821	1,06828	0,9557

Prices according to the generalized model of price indices $p_i = \beta_i p_i^0$

Sector	1	2	3	4	5	6
p_i	1,22982	0,9255	1,24765	0,92821	1,06828	0,9557

Appendix A5. Comparison of Calculations Equilibrium Prices by Price Models and Models of Price Indices Initial Data (Theoretical Packet)

Monetary output in the basic state x_i^0 (table 1)

Sector	1	2	3	4	5	6
x_i^0	43910	1079446	245606	540063	692487	508918

Physical output in the basic state $\left|\bar{x}_i^0\right| = \left|x_i^0\right|$, $i = \overline{1,n}$

Monetary output in the current state, Appendix A4

Sector	1	2	3	4	5	6
x_i	37801	1298739	214502	651677	517841	632283

Physical output in the current state, Appendix A4

Sector	1	2	3	4	5	6
$\bar{x}_i$	30737	1403280	171924	702082	484741	661593

The results of calculations and their comparison

Equilibrium prices by model $(I-S)p=\mu$ (exact model on the output balance)

Sector	1	2	3	4	5	6
p_i	1,22982	0,9255	1,24766	0,92821	1,06828	0,9557

Equilibrium prices according to the generalized model of price indices $p_i = \beta_i p_i^0$

Sector	1	2	3	4	5	6
p_i	1,22982	0,9255	1,24766	0,92821	1,06828	0,9557

Equilibrium prices by model $(I-A')p=\gamma$ according to the input balance and their errors $\Delta p_j = p_i - p_j$, $i=j$

Sector	1	2	3	4	5	6
p_j	1,0055	1,00509	1,27425	1,00562	1,01334	1,00666
Δp_j	0,22432	-0,0796	-0,0266	-0,07741	0,05494	-0,051
Δp_j, %	18,24	-8,6	-2,13	-8,34	5,1	-5,33

Equilibrium prices according to Leontief price model $p_i = \beta_i p_i^0$ and their errors $\Delta p_j = p_i - p_j$, $i=j$

Sector	1	2	3	4	5	6
p_j	0,86085	1,20322	0,87334	1,20668	0,7478	1,24241
Δp_j	0,36897	-0,27772	0,37432	-0,27847	0,32048	-0,28671
Δp_j, %	30	-30	30	-30	30	-30

Appendix A6. Comparison of Calculations Equilibrium Prices by Price Models and Models of Price Indices Initial Data (Realistic Packet)

Output in units of output $\bar{x}_i$

Sector	1	2	3	4	5	6

$\bar{x}_i$	28823	181510	113114	126903	135156	155340

Added value v_j – Appendix A3

Final demand f_i – Appendix A3

Monetary output x_i – Appendix A2, A3

The results of calculations and their comparison

Equilibrium prices according to the model $p = (I - S)^{-1}\mu \ \left(p_i = x_i / \bar{x}_i\right)$

Sector	1	2	3	4	5	6
p_i	1,31148	7,15519	1,896335	5,13524	3,83142	4,07032
Imbalance $\delta x_i = x_i - p_i \bar{x}_i$	0	0	0	0	0	0

Estimated indicators, price indices β_i, equilibrium prices $p_i = \beta_i p_i^0$ according to the generalized model of price indices

Sector	1	2	3	4	5	6
$\Delta x_i = x_i - x_i^0$	-6109	219293	-31104	111614	-174646	123365
$\Delta \bar{x}_i = \bar{x}_i - \bar{x}_i^0$	-15087	-897396	-132492	-413160	-557331	-353578
$1 + \Delta x_i / x_i^0$	0,86087	1,20315	0,87336	1,20667	0,7478	1,24241
$1 + \Delta \bar{x}_i / \bar{x}_i^0$	0, 656411	0,168151	0,460551	0,234978	0,195175	0,305236
β_i	1,31148	7,155176	1,896337	5,13525	3,83143	4,07033
p_i	1,31148	7,15518	1,89634	5,13525	3,83143	4,07033

Equilibrium prices by price model $(I - A')p = \gamma$ according to the input balance and their errors $\Delta p_j = p_i - p_j,\ i = j$

Sector	1	2	3	4	5	6
p_j	2,84295	6,74186	3,43239	5,3281	3,71639	4,40406
Δp_j	-1,53147	0,41332	-1,53605	-0,19285	0,11504	-0,33373
Δp_j, %	-116,8	5,8	81	-3,8	3	-8,2

Equilibrium prices according to Leontief price model $p_i = \beta_i p_i^0$ and their errors $\Delta p_j = p_i - p_j,\ i = j$

Sector	1	2	3	4	5	6
p_j	0,86085	1,20322	0,87334	1,20668	0,7478	1,24241
Δp_j	0,45063	5,952	1,023	3,9286	3,08363	2,82792
Δp_j, %	34,4	83,2	53,9	76,5	80,5	69,5

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