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# A Game of boxed pigs to allow robbing food

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**Abstract:** In this short paper, a new simple system of boxed pigs with three troughs, three dispensers and two panels is given. The main result is that each pig will enjoy his own labor if the pressing cost is lower; the big one will enjoy his own labor but his food has to be robbed by the small one if the pressing cost is higher but the big pig's labor can bring profit for him; and none presses his own panel if the big pig's labor brings loss for him. Finally, an example shows an application of the theory in technology development.

**Keywords:** Robbing Food, Boxed Pigs, Running Speed, Eating Speed, Pure Nash Equilibrium

## 1. Introduction

Based on the references [1-4], an axiomatic theory of boxed pigs with one trough at an end and one panel at the other end in a Skinner Box was given (See [5-9]). This system can describe the situation that a laborer can not enjoy first his (or her) own work achievement. In other words, it is allowed that a lazybones at first enjoys a laborer's work achievement. However, in a social system, it is a common phenomenon that a laborer first enjoys his/her own work achievement and a non-laborer is allowed to enjoy the laborer's work achievement after a period of time. But when both labor at the time, their total work achievement is equal to sum of each. Now let us give a new model about boxed pigs whose description is the follows.

## 2. Model

In a special pigpen, three troughs are at three vertices A, B and C of an equilateral triangle with the side length $d$ . Two panels are installed on the troughs A and B and one big pig is by one of the two troughs and one small pig the other one. When one of the two pigs presses his panel, the other one either presses his own panel at the same time or comes here to rob the food quickly. If only one presses his panel, $q$ units of food are dispensed into his trough; but if each presses his panel, $2q$ units of food are dispensed into the trough C and then the two pigs go there to rob the food quickly. Let $u_b$ and $u_s$ denote the big pig's and the small pig's running speeds respectively ($u_b \geq u_s$). And suppose $v_b$ and $v_s$ denote the big pig's and the small pig's eating speeds respectively ($v_b > v_s$).We suppose the big pig can eat b, t and s units when he alone presses his own panel, when the two pigs press their own panels together, and when the small pig alone presses his own panel, respectively. Pressing his panel needs pay $c$ units of cost where $c < b$ , but running to rob the food needs not pay any cost. Then the game can be written as

$$
\begin{array}{ccc}
 & & \text{Small Pig} \\
 & & \begin{array}{cc} \text{Press} & \qquad\text{Rob} \end{array} \\
\text{Big Pig} & \begin{array}{c} \text{Press} \\ \text{Rob} \end{array} & \begin{bmatrix} (t-c, 2q-t-c) & (b-c, q-b) \\ (s, q-s-c) & (0,0) \end{bmatrix}.
\end{array}
$$

## 3. Theorems

We shall give some theorems without proof in this section.

Theorem 1 (The big pig's incomes) *The big pig's incomes are the follows when he alone presses his own panel, when the two pigs press their own panels together, and when the small pig alone presses his own panel, respectively.*

$$b=\frac{(qu_s+dv_s)v_b}{u_s(v_b+v_s)},\ t=\frac{2v_bq}{v_b+v_s}+\frac{(u_b-u_s)v_bv_sd}{u_bu_s(v_b+v_s)},$$

$$s=\frac{(u_bq-v_sd)v_b}{u_b(v_b+v_s)},\ (d<\frac{u_sq}{v_b}).$$

Proof: (1) When the big pig presses its own panel alone, the small one needs the time $d/u_s$ to arrive at the trough. By this time, the big one has eaten food $v_bd/u_s$ units and so remains $q-v_bd/u_s$ units within the trough. Thus the big pig can eat

$$b=\frac{v_bd}{u_s}+\frac{v_b}{v_b+v_s}(q-\frac{v_bd}{u_s})=\frac{(u_sq+v_sd)v_b}{u_s(v_b+v_s)}.$$

(2) When the two pigs press their panel at the same time, $2q$ units of food would be dispended into the trough C. Since $u_b\ge u_s$, the big one first arrives there. When the small one arrives there too, the big one has eaten $v_b(d/u_s-d/u_b)$ units and so remains $2q-v_b(d/u_s-d/u_b)$ units within the trough. Thus the big one can eat

$$t=v_b(\frac{d}{u_s}-\frac{d}{u_b})+\frac{v_b}{v_b+v_s}[2q-v_b(\frac{d}{u_s}-\frac{d}{u_b})]=\frac{2v_bq}{v_b+v_s}+\frac{(u_b-u_s)v_bv_sd}{u_bu_s(v_b+v_s)}$$

(3) When the small one presses his own panel alone, it has eaten $v_sd/u_b$ units and so remains $q-v_sd/u_b$ units within his trough. Therefore the big pig can eat

$$s=\frac{v_b}{v_b+v_s}(q-\frac{v_sd}{u_b})=\frac{(u_bq-v_sd)v_b}{u_b(v_b+v_s)}.$$

(4) It can be obtained that $d<u_sq/v_b$ by that $q>b=\dfrac{(qu_s+dv_s)v_b}{u_s(v_b+v_s)}$. Q.E.D.

Theorem 2 (Basic Inequalities) We have the inequality $\max\{s,\gamma\}<b<q<t=b+s$, where

$$\gamma=\frac{(u_bq+v_bd)v_s}{u_b(v_b+v_s)}$$

Proof: It can be proved that $b+s=t$. By Theorem 1(4), we have $d<u_sq/v_b<u_bq/v_s$. Thus

$$s=\frac{(u_bq-v_sd)v_b}{u_b(v_b+v_s)}>0,\ b-s=\frac{(u_b+u_s)v_bv_sd}{u_bu_s(v_b+v_s)}>0,$$

$$t-q=\frac{u_bu_s(v_b-v_s)q+(u_b-u_s)v_bv_sd}{u_bu_s(v_b+v_s)}>0,$$

and $b-\gamma=\dfrac{u_bu_s(v_b-v_s)q+(u_b-u_s)v_bv_sd}{u_bu_s(v_b+v_s)}>0$.

Therefore the inequalities we need are proved. Q.E.D.

Theorem 3 (Set of Pure Nash Equilibria) *The set of pure Nash equilibria is*

$$\mathrm{PNE}(\Gamma)=\begin{cases}\{(\text{Press},\text{Press})\}, & c<\gamma,\\ \{(\text{Press},\text{Press}),(\text{Press},\text{Rob})\}, & c=\gamma,\\ \{((\text{Press},\text{Rob})\}, & \gamma<c<b,\\ \{(\text{Press},\text{Rob}),(\text{Rob},\text{Rob})\}, & c=b,\\ \{(\text{Rob},\text{Rob})\}, & c>b.\end{cases}$$

Proof: Based on Theorem 1, the following equalities are clear.

$$t-c-s=b-c=\frac{u_sv_bq+v_bv_sd-u_s(v_b+v_s)c}{u_s(v_b+v_s)},\text{ and}$$

$$2q-t-c-(q-b)=q-s-c=\frac{u_bv_sq+v_bv_sd-u_b(v_b+v_s)c}{u_b(v_b+v_s)}.$$

We will divide the proof into the five cases as follows.
Case 1. $c<\gamma$. Since

$t-c-s=b-c>0$, and $2q-t-c-(q-b)=q-s-c>0$,

we can, by scribing method, obtain that

$$\begin{array}{ccc} & \text{Press} & \text{Rob}\\ \begin{array}{c}\text{Press}\\ \text{Rob}\end{array} & \multicolumn{2}{c}{\begin{bmatrix}(\underline{t-c},\underline{2q-t-c}) & (\underline{b-c},q-b)\\ (s,\underline{q-s-c}) & (0,0)\end{bmatrix}}\end{array}.$$

Thus $\mathrm{PNE}(\Gamma)=\{(\text{Press},\text{Press})\}$.
Case 2. $c=\gamma$. Since

$t-c-s=b-c>0$, and $2q-t-c-(q-b)=q-s-c=0$,

we have

$$\begin{array}{ccc} & \text{Press} & \text{Rob}\\ \begin{array}{c}\text{Press}\\ \text{Rob}\end{array} & \multicolumn{2}{c}{\begin{bmatrix}(\underline{t-c},\underline{2q-t-c}) & (\underline{b-c},\underline{q-b})\\ (s,\underline{q-s-c}) & (0,\underline{0})\end{bmatrix}}\end{array}.$$

Thus $\mathrm{PNE}(\Gamma)=\{(\text{Press},\text{Press}),(\text{Press},\text{Rob})\}$.

The other cases can be easily proved by the similar methods and so we will omit them. Q.E.D.

Theorem 2 tells us that each pig will earn its own living if the labor cost is lower; the big one will work and the small rob the big one's work achievement if the cost is higher but the big pig working can brings him profit; none works if the big one's work brings loss for it.

It can not be guaranteed by Jiang (2010) that this game has a completely pure Nash equilibrium because

$$(a_{00}-a_{10})+(a_{11}-a_{01})=t-c-s-b+c=t-s-b=0,$$
$$(b_{00}-b_{01})+(b_{11}-b_{10})=-t+b+s=0,$$

where

$$\begin{bmatrix}(t-c,2q-t-c) & (b-c,q-b)\\ (s,q-s-c) & (0,0)\end{bmatrix}=\begin{bmatrix}(a_{00},b_{00}) & (a_{01},b_{01})\\ (a_{10},b_{10}) & (a_{11},b_{11})\end{bmatrix}.$$

# 4. An Example of Applications

In a region, there are a big factory and a small factory and there is a generic project. If one factory develops it, then the other one has to imitate it after the success. In this case the new products can bring the total income \$100 million; if the two factories develop it independently at the same time, then the total income is \$200 million. The big factory's and the small factory's imitation abilities are 2 time units and 1 time unit of development workload per month respectively. The big factory's and the small factory's sales abilities are \$4 million and \$3 millions per month respectively. Based on the confidentiality of their technologies, there are the advantages and disadvantages between their products. However the competition makes their have to imitate each other and learn from the other's strong points to offset his weaknesses. Thus we can assume the total imitation workload is fixed whether one factory develops it or two. We ask: (1) what should the total imitation workload be? (2) If the total imitation workload is 24 time units, what are the two factories' incomes when each of them develops it independently? (3) If the total imitation workload is 24 time units, how mach is the development cost if both develop it or the big one does it alone?

Solution: We have known that $q=100$, $u_b=2$, $u_s=1$, $v_b=4$, and $v_s=3$.

(1) By theorem 1, it can be obtained that $d<u_s q/v_b=25$. This tells us that the total imitation workload is less than 25 time units.

(2) Since $d=24$, by theorem 2, we obtain that

$$b=\frac{(u_s q+v_s d)v_b}{u_s(v_b+v_s)}\approx 98.29, \text{ and } s=\frac{(u_b q-v_s d)v_b}{u_b(v_b+v_s)}\approx 36.57.$$

This result shows that the sole big factory's development can bring the income \$ 98.29 millions for him and the small one's \$63.43. Since $t=b+s=134.86$, the big factory's income is \$134.86 millions and the small one's \$65.14 if both develop it at the time.

(3) Since $d=24$ and $\gamma=\frac{(u_b q+v_b d)v_s}{u_b(v_b+v_s)}\approx 53.14$, by theorem 3, the both will develop it if the development cost is less than \$53.14 millions and the big factory will develop it and the small will wait for imitation if the development cost is between \$ 53.14 and \$ 98.29 millions.

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